Perfectly matched layers for the linear Boltzmann equation

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The linear Boltzmann equation is a hyperbolic integro-partial differential equation that describes the dynamics of a single-particle probability distribution in location-velocity phase space. The dynamics is governed by streaming, damping and scattering. The two main challenges in the numerical approximation of solutions to the linear Boltzmann equation are (i) the high dimensionality of the phase space, and (ii) the non-smoothness of the solution.

The linear Boltzmann equation is equivalent to a mixed variational problem that incorporates boundary conditions naturally [1, 2]. However, the natural inclusion of boundary conditions introduces a non-smooth coupling of spatial and velocity variables, which is inconvenient for practical implementations. To overcome this difficulty, we introduce an absorbing layer and consider a perturbed problem such that the resulting variational problems exhibit a tensor-product structure.

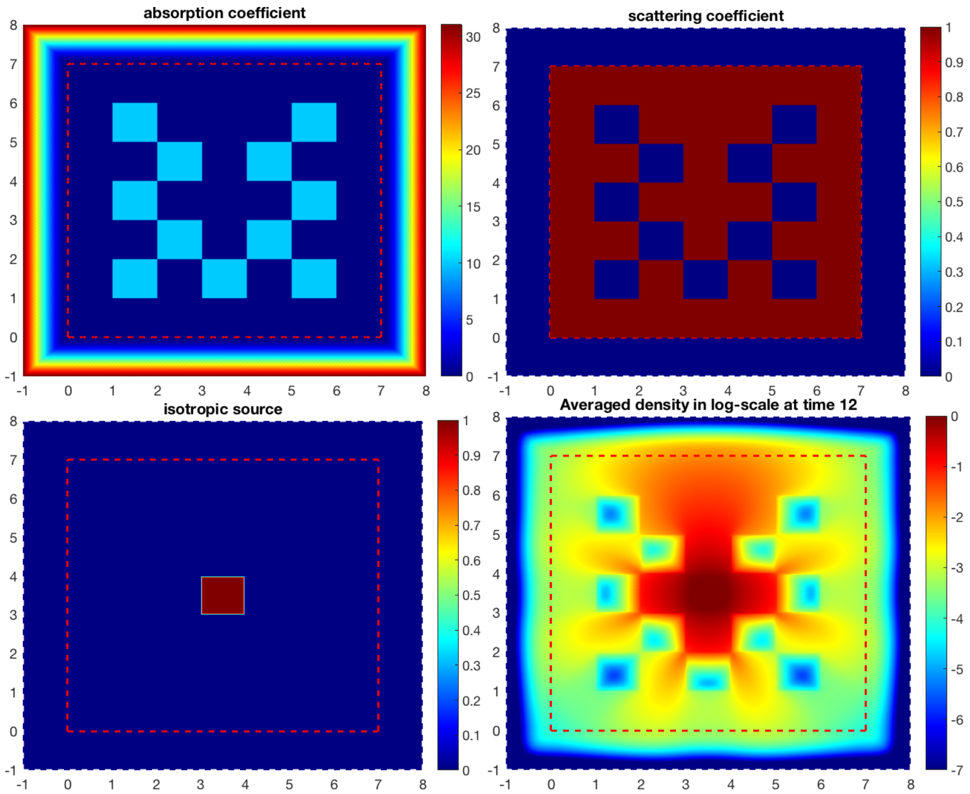

Figure 1: Simulation of a lattice problem imitating the core of a nuclear reactor. The physical domain of interest is $[0, 7] \times [0, 7]$, which is surrounded by a absorbing layer of thickness 1 (marked by the red dotted line). Top row: absorption (left) and scattering (right) parameters. Bottom row: Source term (left) and velocity average of a P_{51} approximation of the solution (right).

References

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