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"On optimality of interpolation-based low-rank approximations of large-scale Lyapunov equations"

Abstract:
In this talk, we will discuss projection-based approximations for the solution of Lyapunov equations of the form

$$AXE^T + EXA^T + BB^T = 0,$$

with $A = A^T, E = E^T \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. Recently, in Vandereycken and Vandewalle (2010) a relation between minimizing the objective function

$$f : \mathcal{M} \to \mathbb{R},\ X \mapsto \operatorname{tr}\left(XAXE + BB^T\right)$$

on the manifold $\mathcal{M}$ of symmetric positive semi-definite matrices of rank k and the $\mathcal{L}-$norm defined by the operator

$$\mathcal{L} := E \otimes A + A \otimes E$$

together with the inner product $\langle u, v \rangle_{\mathcal{L}} = \langle u, \mathcal{L}v \rangle$ has been shown. While there the minimization problem was solved by means of a Riemannian optimization approach, here we will discuss an interpolation-based method which leads to the same results but relies on projecting the original Lyapunov equation onto a smaller subspace. It will turn out that this can be achieved by the iterative rational Krylov algorithm (IRKA) or, alternatively, by the alternating direction implicit iteration (ADI) if appropriate shifts are used. Besides a generalization for the case of $A \neq A^T$, we will also discuss an extension for more general equations of the form

$$AXE^T + EXA^T + \sum_{i=1}^{m} N_i X N_i^T + BB^T = 0,$$

with $A = A^T, E = E^T, N_i = N_i^T \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. This is joint work with Peter Benner.