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"The 2D-case: where tensors meet Lyapunov equations"

Abstract:
The 2D tensor (or matrix) case has always been of special interest. We present a framework of preconditioned low-rank Krylov-subspace methods like FGMRES [1] or CG [2] for solving Lyapunov matrix equations involving a rank truncation strategy. As natural preconditioner we chose the LRCF-ADI [3] method. As an application we give examples from Balanced truncation model order reduction for electrical circuits [4] where solving generalized, projected Lyapunov equations is the key task of the underlying algorithm.
Furthermore we want to consider the heat equation in 3 spatial dimensions. In model reduction this leads to a Lyapunov equation with the matrix arising from finite difference discretization in 3 dimensions. Treating this as a linear equation the dimension grows like $\mathcal{O}(n^6)$ which is not feasible. Tackling this problem with our low-rank Krylov-subspace methods we are able to solve this in reasonable time.
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References

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