Preconditioned low-rank methods for high-dimensional elliptic PDE-eigenvalue problems

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We consider elliptic PDE-eigenvalue problems on a tensorized domain, discretized such that the resulting matrix eigenvalue problem $Ax = \lambda x$ exhibits Kronecker product structure. In particular, we are concerned with the case of high dimensions, where standard approaches to the solution of eigenvalue problems fail due to the exponentially growing degrees of freedom. Recent work shows that this curse of dimensionality can be addressed in many cases by approximating the desired solution vector x in a low-rank tensor format. We use the hierarchical Tucker decomposition to develop a low-rank variant of LOPCG, a classical preconditioned eigenvalue solver. Alternatively, the ALS and MALS (DMRG) methods known from computational quantum physics could be applied in this setting.

The ALS and MALS methods require the solution of reduced eigenvalue problems involving $2-4$ dimensional tensors. The standard LOPCG method was recently proposed for the solution of these problems. We propose the low-rank variant of the LOPCG method for the case of the MALS method. Additionally, given a preconditioner of the original system, a preconditioner can be found for the reduced eigenvalue problem. We present a number of numerical experiments, and briefly discuss different choices of stopping criteria and truncation ranks for the reduced eigenvalue problems.