FFT and cross interpolation schemes for QTT format

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Tensor train (TT) format has been recently proposed in [6] as a new method of data-sparse representation of the high-dimensional data (d–*tensors*, i.e. arrays with d indices) based on separation of variables. TT format has the advances of both canonical and Tucker formats: the number of parameters does not grow exponentially with dimension, decomposition/approximation problem is stable and can be computed by methods based on SVD. Using the quantization idea proposed in [5, 4], we can apply TT format to the data of low dimension, resulting in the QTT format.

Recent progress in the development of the efficient algorithms for QTT data includes the FFT algorithm and cross interpolation scheme. The m–dimensional Fourier transform of an $n \times \ldots \times n$ vector with $n = 2^d$ has logarithmic complexity, $O(m^2 d^2 r^3)$, where r is the maximum QTT rank of input, output and all intermediate vectors during the procedure. For vectors with moderate r and large n and m the proposed algorithm outperforms the $O(n^m \log n)$ FFT algorithm of almost volume complexity.

In this talk we describe the class of vectors with $r = 1$. Also, we show how we can combine cross interpolation algorithm with FFT to compute Fourier images of some one–, two– and three–dimensional functions efficiently, as well as the convolutions. The proposed cross interpolation algorithm is based on the DMRG scheme and uses maximum-volume principle [2, 3] to select the interpolation positions. It can be also formulated using the Wedderburn rank reduction framework, applied to 3-tensors in [1].

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