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Title:
Hierarchical Tensor Methods for PDEs with Stochastic Parameters

Abstract:
We consider the problem to solve a (stochastic) parameter dependent equation

$$A(\omega)u(\omega) = b(\omega), \qquad \omega \in \Omega$$

for systems A governed by partial differential operators that depend on ω. Our aim is to calculate quantities of interest (mean, variance, maximum etc.) of the set of solutions. One way to solve such a problem is by expansion of the system, the right-hand side as well as the solution in independent uncorrelated stochastic variables $\omega_1, \ldots, \omega_p$, and then solve the arising large-scale deterministic problem

$$A(\omega_1, \ldots, \omega_p)u(\omega_1, \ldots, \omega_p) = b(\omega_1, \ldots, \omega_p).$$

An alternative approach is to use (quasi or multilevel) Monte Carlo (MC) methods which require just a simple sampling (M simulations), but these are only useful for certain quantities of interest (e.g. the mean). We will present a new approach based on hierarchical Tucker (HT) representations of tensors. This method is based on standard PDE solvers for deterministic systems. The set of solutions is approximated in a low rank (HT) tensor format that allows for many parameters (thousands), since for fixed rank the complexity depends only linearly or quadratically on the number of parameters.