Tensor Decompositions and new models in Spectral theory

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Spectral theory studies the unitary invariants of operators. However it is notorious that a satisfactory analysis including inverse problems of spectral (and scattering) theory is available only in the case of one-dimensional operators. In particular, in the case of discrete one-dimensional operators, which are the matrices, the spectral invariants are the eigenvalues, and every approximation (including rank approximations) is intimately related to the spectrum. Hence, a reasonable attempt to generalize the approximations to multidimensional operators has to be related to a generalization of the notion of spectrum.

We provide some new Tensor decompositions and approximations which are related to new "pseudo-positive models" in Spectral theory of multidimensional operators, cf. [1]. For these models we show that one may define in a natural way the notions of "Determinant" and "Rank" which differ from other approaches, cf. e.g. [2]. The main issue is the notion of (multidimensional) Distributed Spectrum for the operators arising through such models. Based on this notion of Distributed Spectrum, one may introduce new approach to the rank-type approximation of the operators, in particular to the discrete versions which are called tensors. In the one-dimensional case one has an approximation which corresponds to the Gauss-Jacobi measure related to the orthogonal polynomials generated by the spectral measure. Respectively, for our "pseudo-positive models" we are able to generalize the usual rank approximation analogous to the Eckart-Young one, as well as an analog to the Gauss-Jacobi approximation.

Alternatively, the above models provide a generalization to the multidimensional case of the Jacobi matrix representation, known for matrices (cf. [3], [4]), and hence a new approximation to the multi-way matrices (tensors).

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[2] I. Gelfand, Kapranov, Zelevinsky, Discriminants, Resultants, and Multidimensional Determinants, Birkhaeuser, 1992, 2008.

[3] N. Achiezer, The classical moment problem, Holden Day, 1966. (translation from Russian).

[4] B. Simon, Szegö's Theorem and Its Descendants: Spectral Theory for L_2 Perturbations of Orthogonal Polynomials, Princeton University Press, 2010.