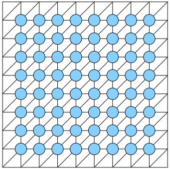
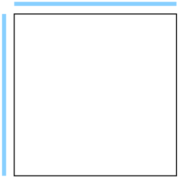
Alternative Clustering for Sparse Matrices

Clustering

Domain:



Matrix:



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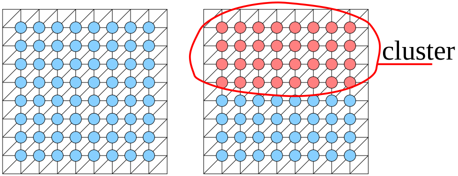
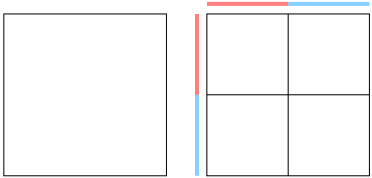


Matrix:



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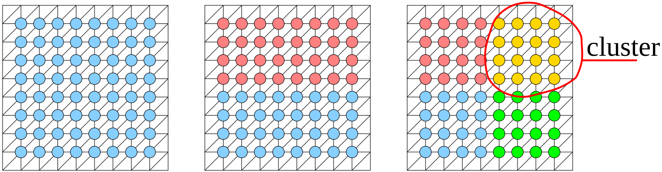

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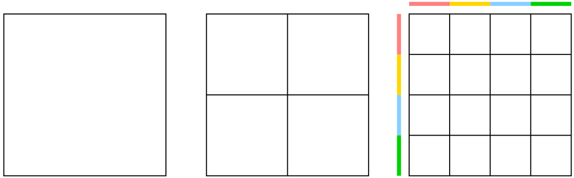

CLUSTERING

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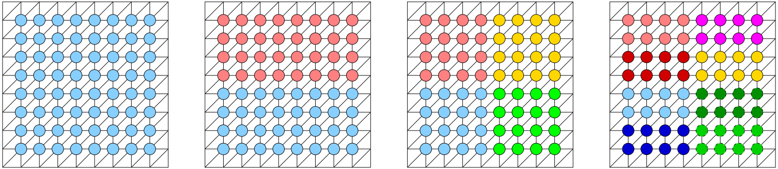

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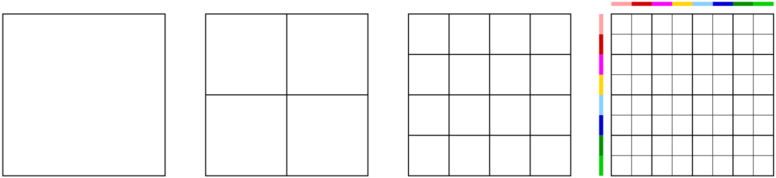

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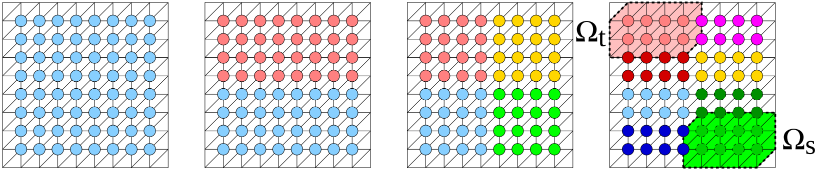
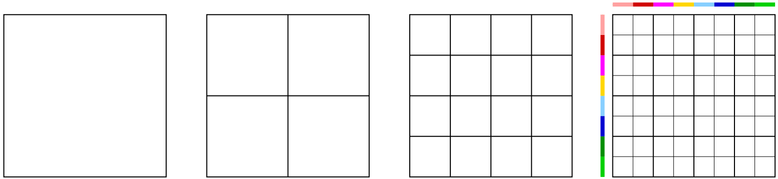


Matrix:



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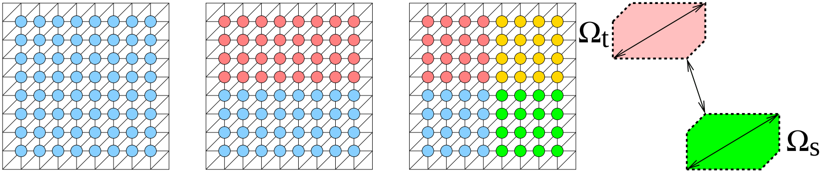

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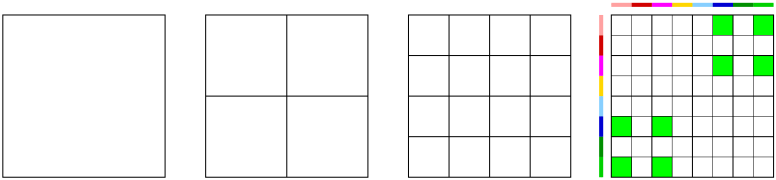

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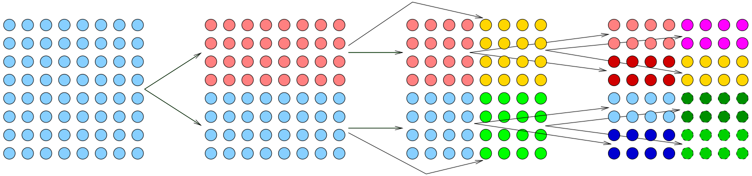
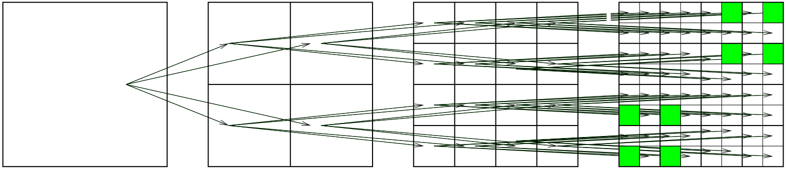
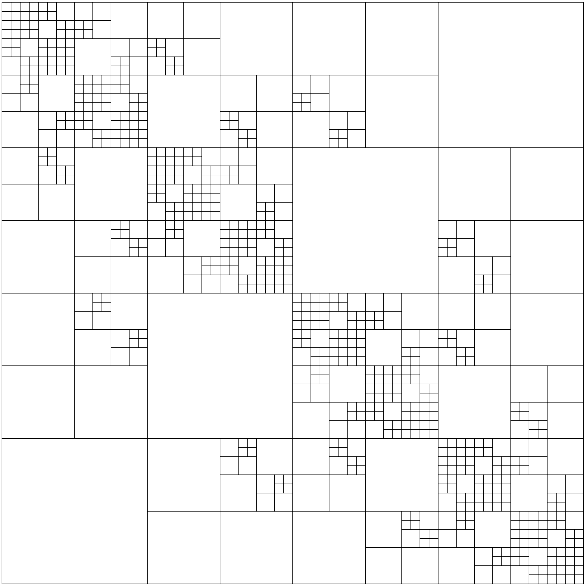


Matrix:

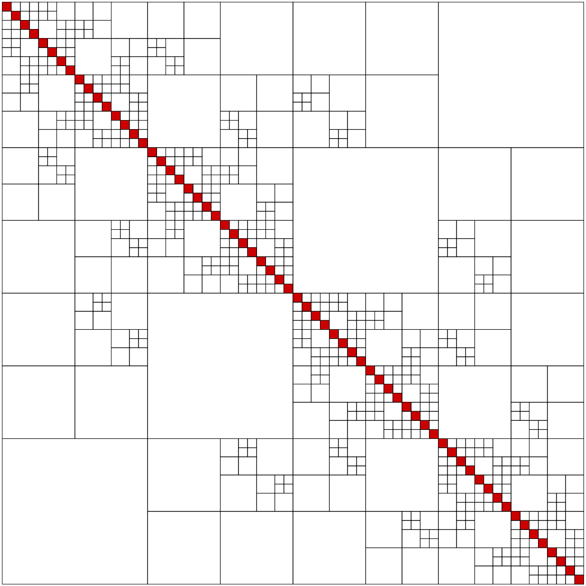


MATRIX STRUCTURE



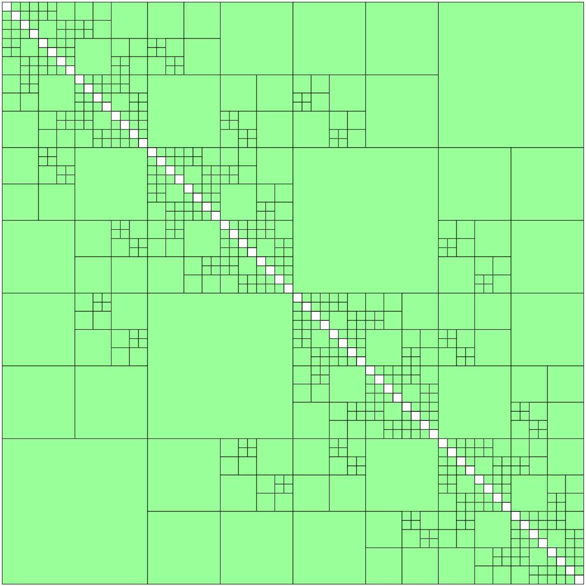
- $\mathcal{O}(n)$ blocks

Matrix Structure



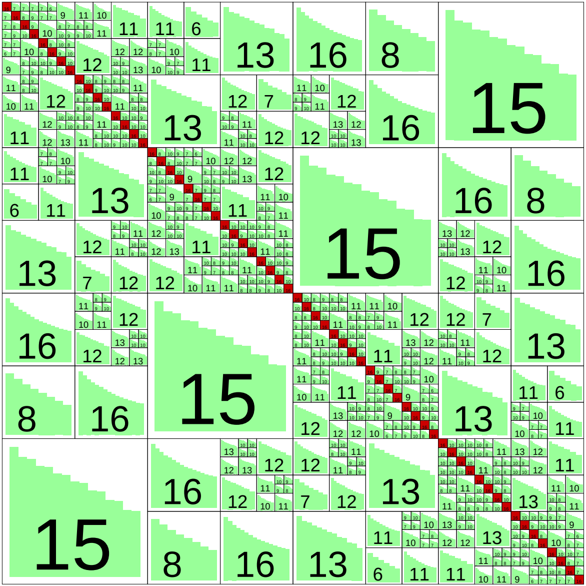
- $\mathcal{O}(n)$ blocks
- Small red blocks: `fullmatrix`

Matrix Structure



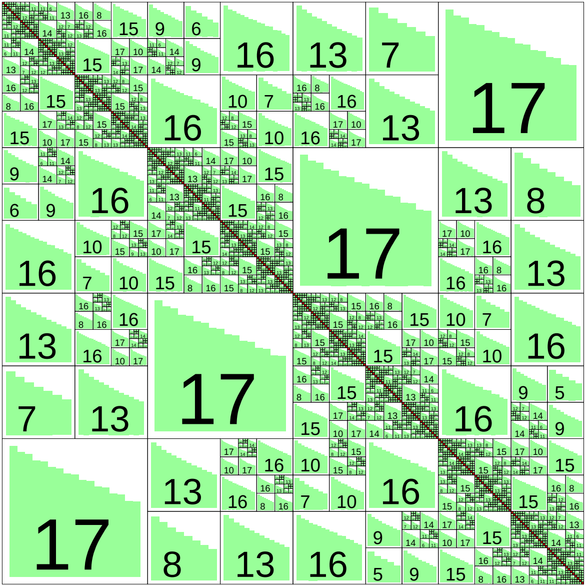
- $\mathcal{O}(n)$ blocks
- Small red blocks: `fullmatrix`
- All other blocks: `rkmatrix`

Matrix Structure



- blockwise: exponential decay of singular values
- matrix size: 1024×1024 3.2 MB

MATRIX STRUCTURE

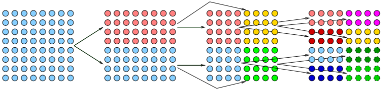


- blockwise: exponential decay of singular values
- matrix size: 4096×4096 20.7 MB

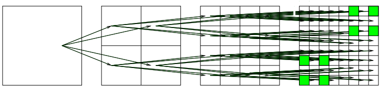
$\mathcal{H}$-Matrix Arithmetic

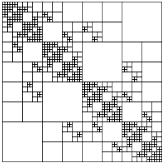
How do we calculate with $\mathcal{H}$-matrices ?

1. Construct **cluster tree** T_I
   → geometry information
   → black box (`sparsematrix` graph)



2. Construct **block cluster tree** $T_{I\times I}$
   → partition of matrix
   → requires admissibility condition





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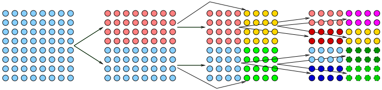

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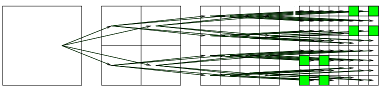

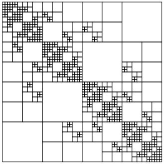

3. Convert `sparsematrix` A into $\mathcal{H}$-Matrix H

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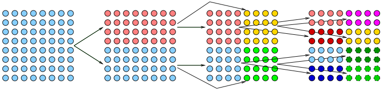

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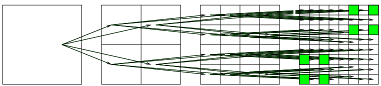
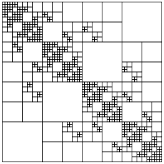


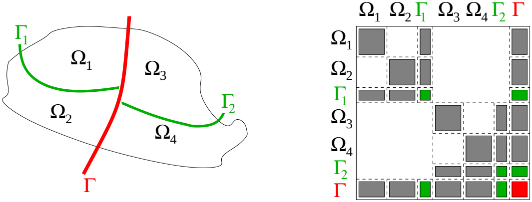


3. Convert `sparsematrix` A into $\mathcal{H}$-Matrix H
4. Compute $\mathcal{H}$**-LU** factorization

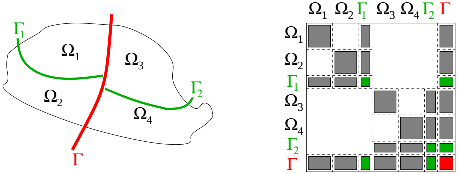
Result:

$$\mathbf{A} \approx \mathbf{L}\,\mathbf{U}, \qquad L, U \in \mathcal{H}(T_{I\times I})$$

Domain Decomposition & Nested Dissection



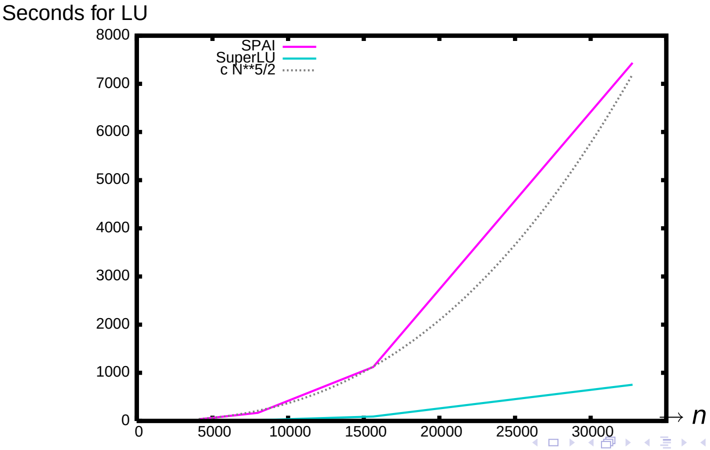
Nested dissection and matrix structure



matrix structure of LU-factorization

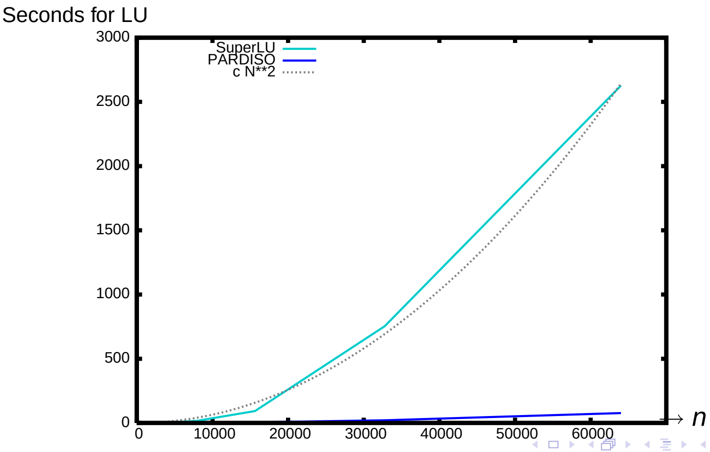
Domain Decomposition & Nested Dissection

- SPAI (Barnard/Grote/Broeker/Hagemann)
- SuperLU (Demmel/Gilbert/Li)



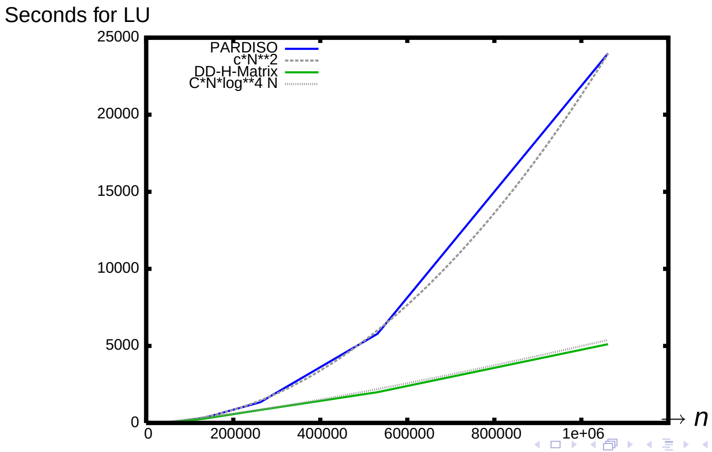
Domain Decomposition & Nested Dissection

- SuperLU (Demmel/Gilbert/Li)
- PARDISO (Schenk/Gärtner/Fichtner)



DOMAIN DECOMPOSITION & NESTED DISSECTION

- PARDISO (Schenk/Gärtner/Fichtner)
- DD-H-Matrix (Grasedyck/Kriemann/LeBorne)



Domain Decomposition & Nested Dissection

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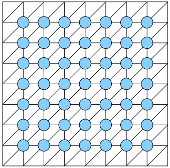
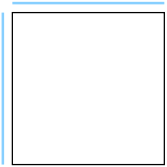


Matrix:



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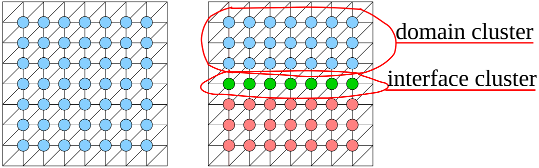

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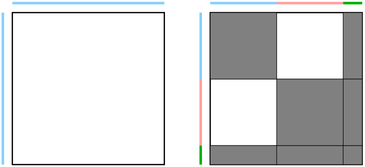

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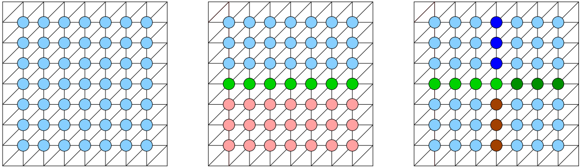

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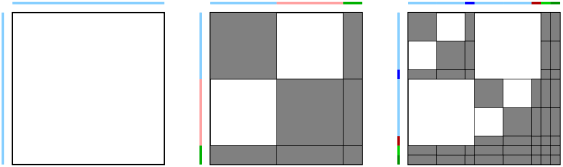

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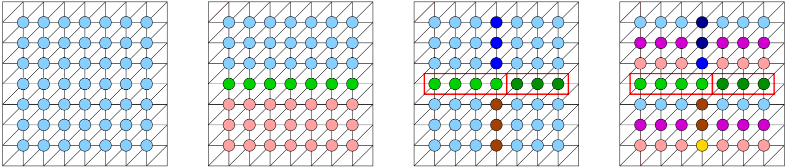

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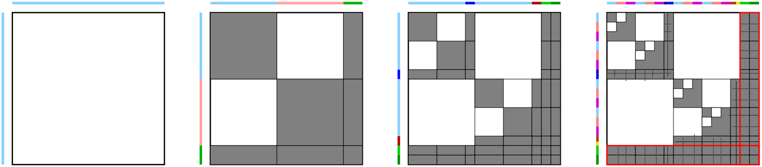
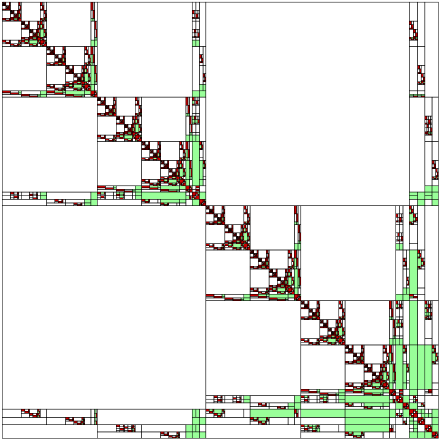
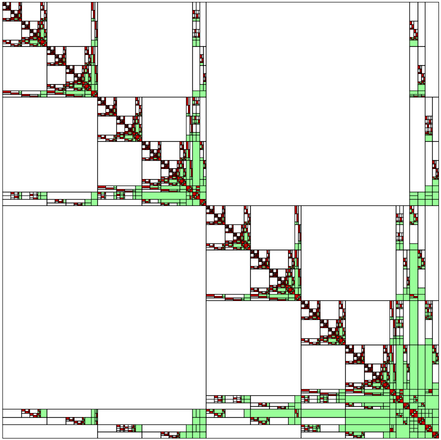


Domain Decomposition & Nested Dissection

Example ($d = 3$)

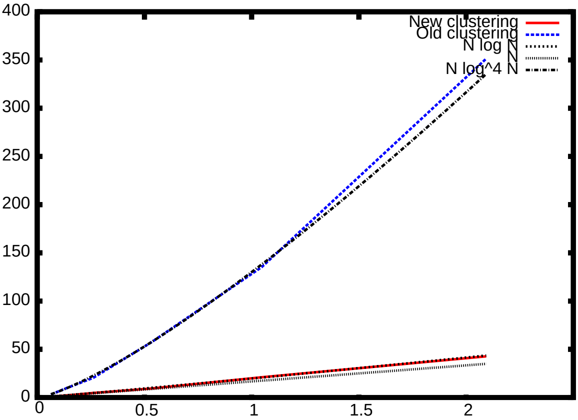


Matrix A.



$\mathcal{H}$-LU decomposition

OLD AND NEW CLUSTERING



$\mathcal{H}$-LU versus DD-$\mathcal{H}$-LU for 2d-Laplace

$\mathcal{H}^2$-MATRICES

$\mathcal{H}$-Matrices:

1. Storage: $\mathcal{O}(N \log(N) k)$
2. LU: $\mathcal{O}(N \log^2(N) k^2)$
3. Rank $k \sim \log^{d-2}(1/\varepsilon)$
4. Total $\mathcal{O}(N \log^{2d-2}(N))$

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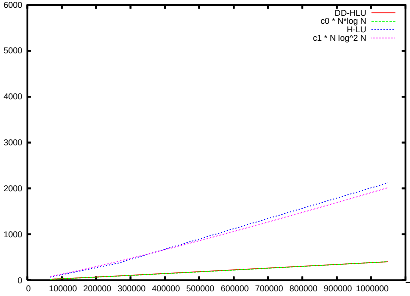
Seconds for LU



$\longrightarrow N$