# Geometrische Mechanik

1. What are the possible types of linear Hamiltonian systems with one degree of freedom? How do the corresponding Hamiltonian functions look like? Which of the occurring equilibria are structurally stable — and in what sense?

2. A result of this exercise turns out to be that

$$\mathfrak{sl}_n(\mathbb{R}) \quad = \quad \{A \in M_{n\times n} \mid \operatorname{tr} A = 0\}$$

is the Lie algebra of the Lie group

$$SL_n(\mathbb{R}) \quad = \quad \{S \in M_{n\times n} \mid \det S = 1\} \ .$$

To this end compute for a curve

$$\begin{array}{rccc} \gamma \ : & \mathbb{R} & \longrightarrow & M_{n\times n} \\ & t & \mapsto & \gamma(t) \end{array}$$

the corresponding tangent vector

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\gamma(t)\right|_{t=0}$$

and use the curve $\gamma(t) = \exp tA$ to conclude that $\mathfrak{sl}_n(\mathbb{R}) = T_{\mathrm{Id}} SL_n(\mathbb{R})$. What are the consequences for the flow of a divergence-free vector field?

3. Classify all linear Hamiltonian systems in two degrees of freedom that have no multiple eigenvalues. *Hint:* if $\lambda$ is an eigenvalue, then not only $\bar{\lambda}$ but also $-\lambda$ is an eigenvalue as well (as is $-\bar{\lambda}$).