Geometrische Mechanik

9. Verify that the triple product

$$\{f, g\} \;=\; \langle \nabla f \times \nabla g \mid \nabla S_a^b \rangle$$

with

$$S_a^b(u, v, w) \;=\; (1 - u^2)\, w \;-\; \frac{(a - bu)^2}{2} \;-\; \frac{v^2}{2}$$

is the (twice) reduced Poisson structure of the Lagrange top.

10. In this exercise we study some properties of the Poisson bracket on a symplectic manifold.

Show that if two functions F, G are integrals of H then so is $\{F, G\}$.

Show that if $h : \mathcal{P} \longrightarrow \mathcal{P}$ is symplectic then $\{F, G\} \circ h = \{F \circ h, G \circ h\}$. What is the meaning of this equation?