

Geometrische Mechanik

15. Show that reduction of the left S^1 -action

$$L : \begin{array}{ccc} S^1 \times (SO(3) \times \mathbb{R}^3) & \longrightarrow & SO(3) \times \mathbb{R}^3 \\ (\rho, (g, \ell)) & \mapsto & (\exp_\rho \circ g, \ell) \end{array}$$

yields the reduced equations of motion

$$\begin{aligned} \dot{\eta} &= \eta \times \nabla_\ell H \\ \dot{\ell} &= \eta \times \nabla_\eta H + \ell \times \nabla_\ell H \end{aligned}$$

where

$$\eta = \begin{pmatrix} \langle e_z | e_1 \rangle \\ \langle e_z | e_2 \rangle \\ \langle e_z | e_3 \rangle \end{pmatrix}$$

denotes the Poisson vector, i.e. the vertical axis expressed in body co-ordinates.

16. Analyse the force free symmetric rigid body, or, equivalently, the Lagrange top with fixed centre of mass.