Geometrische Mechanik

15. Show that reduction of the left S^1–action

$$\begin{array}{rccc} L \ : & S^1 \times (SO(3) \times \mathbb{R}^3) & \longrightarrow & SO(3) \times \mathbb{R}^3 \\ & (\rho, (g, \ell)) & \mapsto & (\exp_\rho \circ g, \ell) \end{array}$$

yields the reduced equations of motion

$$\begin{array}{rcl} \dot{\eta} & = & \eta \times \nabla_\ell H \\ \dot{\ell} & = & \eta \times \nabla_\eta H \ + \ \ell \times \nabla_\ell H \end{array}$$

where

$$\eta \ = \ \begin{pmatrix} \langle e_z \mid e_1 \rangle \\ \langle e_z \mid e_2 \rangle \\ \langle e_z \mid e_3 \rangle \end{pmatrix}$$

denotes the Poisson vector, i.e. the vertical axis expressed in body co-ordinates.

16. Analyse the force free symmetric rigid body, or, equivalently, the Lagrange top with fixed centre of mass.