Adaptive Finite Volume Schemes for Conservation Laws based on Local Multiresolution Techniques

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Abstract. A new adaptive strategy for finite volume schemes is presented based on local multiscale decompositions. By this approach, the complexity of the scheme can be restricted to the number of significant coefficients. For one–dimensional, scalar conservation laws, the efficiency of the concept is verified for several test configurations.

1. Introduction

We consider partial differential equations of the form

$$\frac{\partial\, u(t,x)}{\partial\, t} + \frac{\partial\, f(u(t,x))}{\partial\, x} = 0, \qquad 0 < t < T,\ a < x < b \tag{1}$$

with $T > 0$ and $a, b \in \mathrm{IR}$ fixed, the so–called conservation laws. Herein, $u : [a,b] \to \mathrm{IR}$ is the conservative quantity and $f : \mathrm{IR} \to \mathrm{IR}$ the flux. In order to ensure a unique solution, an initial condition $u(0,x) = u_0(x),\ a < x < b$, has to be imposed. In practical applications, boundary conditions of the form $g_a(u(t,a)) = 0,\ g_b(u(t,b)) = 0,\ 0 \le t \le T$ are also needed. The one–dimensional scalar equation (1) is a simple model for more complex systems arising from fluid dynamics, e.g. Euler equations.

During the last decade, finite volume schemes have been proven to be very robust and reliable approximation schemes for conservation laws as has been verified by numerous applications. However, the construction of robust and efficient schemes is not yet finished. More advanced approaches are based on the heterogeneity of the flow field, i.e., several physical effects proceeding on different spatial and temporal scales have to be appropriately resolved. In order to resolve these effects by a numerical scheme, different discretization lengths are required where the highest resolution is determined by the finest scale. In the case of multidimensional flow fields this will, in general, exceed presently available memory capacities or computational time will become prohibitively long. Therefore, algorithms have to be developed with computational effort and memory capacity proportional to the complexity of the underlying problem, i.e., only in regions where small–scale effects are present the discretization has to be chosen sufficiently fine, and elsewhere a much coarser discretization will be sufficient. In view of realistic simulations of multidimensional flow fields around complex geometries uniform discretizations