Adaptive multiresolution finite volume discretization of the Variational Multiscale Method. General Framework.

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In order to investigate turbulent phenomena in compressible flows the Variational Multiscale method is used. This method is usually applied to incompressible flows. In a first step, we derive the VMS method for the compressible Navier-Stokes equations. The resulting weak formulation of the flow equations is split into resolved and unresolved scales using multiresolution techniques based on biorthogonal wavelets. Since the in principle infinite-dimensional subspace of fluctuations is also discretized, it needs to be stabilized by additional dissipative terms. The compressible VMS method is then incorporated into an adaptive multiresolution finite volume solver, where grid adaptation is also performed by means of the multiresolution analysis.

1. Introduction

The investigation of turbulent phenomena in compressible flows at high Reynolds numbers using Direct Numerical Simulations (DNS) is restricted to simple configurations only, because of the wide range of relevant scales that have to be resolved. In case of more complex applications such as the interaction of a hypersonic flow field with an injection jet of cooling gas through single boreholes or slots, it will not be feasible to resolve all scales. For this purpose low parametric turbulence models seem not to be reliable and capable of dealing with strongly heterogeneous discretizations resulting from locally refined grids. Here recent developments of the Variational Multiscale (VMS) method [1, 2] and related Subgrid Scale Methods [3, 4] seem to be more suited. These methods can be considered as advanced Large Eddy Simulation Methods (LES) [5–8]. Although these methods have been applied almost exclusively to incompressible flows we will apply the VMS method here to compressible flows. The key idea of the VMS method is to use the projection to an ansatz space of given resolution instead of convolution-based smoothing. Then the weak formulation of the flow equations that is split into resolved and unresolved scales shows the influence of fluctuation residual on the large scales and vice versa. The in principle infinite-dimensional subspace of fluctuations is also discretized and, hence, has to be stabilized. For stabilization dissipative terms are used containing model parameters that depend nonlinearly on the gradients of the fluctuations. The choice of the model is not as sensitive as in the context of LES, because it only affects the scales that are of the magnitude of the resolved fluctuation scales. Thus the influence of the model on the entire range of resolved scales is reduced. Another principle advantage of the VMS method in comparison to classical LES approaches lies in the fact that the commutation error resulting from the filter and