Multiscale Concept for Conservation Laws

Birgit Gottschlich-Müller,[1] Siegfried Müller

Institut für Geometrie und Praktische Mathematik
RWTH Aachen, 52056 Aachen

E-Mail: mueller@igpm.rwth-aachen.de or
birgit@igpm.rwth-aachen.de

Abstract

The physical modeling of fluid flows neglecting effects due to diffusion, viscosity and heat conduction results in hyperbolic differential equations of first order describing the change of state in space and time. These equations are the so-called conservation laws. The solution is characterized by the finite speed of propagation and the developing of discontinuities. Stable and robust numerical schemes are based on upwind techniques in order to handle shocks and contact dicontinuities. In general, schemes of this type are very expensive. In 1993 Harten derived a concept to reduce the computational costs. It is based on a multiscale decomposition of the approximation. This decomposition enables him to locate regions with singularities. Near discontinuities Harten applies an expensive upwind scheme and otherwise a much cheaper finite difference scheme of high order.

Harten's multiscale decomposition is based on the primitive function of the solution. Thus, his ansatz is inherently restricted to structured grids. In this paper we present an alternative multiscale decomposition using general reconstruction methods. These techniques can be extended to multidimensional problems even for unstructured grids. Here, we only consider the onedimensional case and we verify that our ansatz coincides with those of Harten's decomposition method. Furthermore, we apply our algorithm to dimensionally reduced hypersonic stagnation point flows around spheres.

Key Words: Multiresolution, conservation laws, finite volume methods, hybrid schemes.

AMS subject classification: 65M12, 76Axx, 35L65

[1]The author is supported by the Deutsche Forschungsgemeinschaft in the Graduiertenkolleg "Transportvorgänge in Hyperschallströmungen" at the RWTH Aachen.