ON MULTI–SCALE CONCEPTS FOR MULTI–DIMENSIONAL CONSERVATION LAWS

Birgit Gottschlich–Müller[1], Siegfried Müller
Institut für Geometrie und Praktische Mathematik
RWTH Aachen
Templergraben 55
52056 Aachen, Germany

SUMMARY

In 1993 Harten developed a concept to reduce the computational costs of finite volume schemes applied to hyperbolic conservation laws. It is based on a multi–scale decomposition of discrete data. By this decomposition singularities in the flow field are detected. In the present work, we introduce a general multi–scale concept for discrete data. It is essentially based on prolongation and restriction matrices as known from multigrid theory. Two different concepts are presented how to determine the transfer matrices. The first one is based on Harten's general framework [$H96$] applied to cell averages. The second strategy is based on bi–orthogonal wavelets examined by Carnicer, Dahmen and Peña [CDP]. The relation between the two concepts is presented.

Numerical calculations for the rotational symmetric Euler equations are presented. We consider the unsteady shock focussing problem. The results show a significant reduction in computational time.

INTRODUCTION

In this paper we consider partial differential equations of the form

$$\frac{\partial\, u(t,\mathbf{x})}{\partial\, t} + \sum_{m=1}^{d} \frac{\partial\, f_m(u(t,\mathbf{x}))}{\partial\, x_m} = s(\mathbf{x}, u(t,\mathbf{x})), \qquad 0 < t < T,\ \mathbf{x} \in \Omega, \tag{1}$$

where $T > 0$ and $\Omega \in \mathrm{IR}^d$ fixed. Systems of this type are called *conservation laws*. Here, $u : \Omega \rightarrow \mathrm{IR}$ is the conservative quantity, $f_m : \mathrm{IR} \rightarrow \mathrm{IR}$ the flux in the m–th coordinate direction and $s : \mathrm{IR} \rightarrow \mathrm{IR}$ the source term. In order to ensure a unique solution, an initial condition

$$u(0,\mathbf{x}) = u_0(\mathbf{x}), \qquad \mathbf{x} \in \Omega, \tag{2}$$

[1]The author is supported by the Deutsche Forschungsgemeinschaft in the Graduiertenkolleg "Transportvorgänge in Hyperschallströmungen" at the RWTH Aachen.