# A Riemann Solver for the Euler Equations with Non–Convex Equation of State

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**Abstract**

An algorithm is presented by which the Riemann problem in gas dynamics can be solved for a *non–convex* equation of state. The non–convexity causes non–standard wave patterns, e.g. *composite* waves and anomalous behavior of physical quantities along classical waves, e.g. *expansion* and *sonic* shocks, may occur. In contrast to standard Riemann solvers, these effects are taken into special account. The derivation of the algorithm is based on construction principles which are motivated by analytical investigations in the context of gas dynamics as well as general systems of hyperbolic conservation laws.

With the aid of the extended Riemann solver we compare an ideal gas and a real gas. The results show that the non–classical effects are significant and can not be treated as a perturbation of an ideal gas. This implies that existing flow solvers have to be appropriately adapted when performing numerical simulations of flows in fluids exhibiting non–classical effects such as retrograde fluids.

**Key Words:** Riemann problem, non–convex equation of state, composite wave, compression wave, expansion shock, sonic shock.

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