Riemann Problem for the Euler Equation with Non-Convex Equation of State including Phase Transitions

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Summary. An exact Riemann solver is developed for the investigation of non-classical wave phenomena in BZT fluids and fluids which undergo a phase transition. Here we outline the basic construction principles of this Riemann solver employing a general equation of state that takes negative nonlinearity and phase transition into account. This exact Riemann solver is a useful validation tool for numerical schemes, in particular, when applied to the aforementioned fluids. As an application, we present some numerical results where we consider flow fields exhibiting non-classical wave phenomena due to BZT fluids and phase transition.

1 Introduction

The dynamics of compressible flows has a strong influence on the design of aircraft and turbomachinery. In many applications the fluid is modeled by a perfect gas. In the range of perfect gas theory only two types of waves are possible and allowed by the entropy inequality, the *compression shock* and the *centered expansion fan* or *rarefaction wave*. A compression shock is a discontinuity where the pressure of the fluid increases while the shock is passing, whereas in a rarefaction wave the pressure decreases and the wave shape forms a fan.

However, the perfect gas model is no longer appropriate when dealing with so-called BZT fluids characterized by a large heat capacity, e.g., high molecular fluorcarbons and hydrocarbons. These fluids exhibit a region in the phase space where the isentropes in the pressure-volume plane are non-convex, see Figure 2. It is also referred to as region of *negative nonlinearity*. From experiments, see [BBKN83], it is known that there may occur non-classical effects such as *expansion shocks* and *compression fans*, as well as waves composed of adjacent shocks *and* rarefaction parts. In short, most of the classical inequalities and effects are reversed. In addition, below the critical point in the phase space, see Figure 2, liquid and vapor co-exist in a so-called