Fully adaptive multiscale schemes for conservation laws employing locally varying time stepping*

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Abstract

In recent years the concept of fully adaptive multiscale finite volume schemes for conservation laws has been developed and analytically investigated. Here the grid adaptation is performed by means of a multiscale analysis based on biorthogonal wavelets. So far, all cells are evolved in time using the same time step size. In the present work this concept is extended incorporating locally varying time stepping. A general strategy is presented for explicit as well as implicit time discretization. It can be applied to a scalar equation and systems of equations for arbitrary space dimensions. For reasons of simplicity, the strategy is developed in detail for one-dimensional problems. The efficiency and the accuracy of the proposed concept is numerically investigated for 1D scalar conservation laws. First 2D Euler computations verify that it can also be applied to multidimensional systems.

Key words. Multiscale techniques, local grid refinement, locally varying time stepping, finite volume schemes, conservation laws.

AMS subject classification. 41A58, 65M50, 65M12

1 Introduction

The solution of hyperbolic conservation laws typically exhibits locally steep gradients and large regions where it is smooth. To account for the highly nonuniform spatial behavior, we need numerical schemes that adequately resolve the different scales, i.e., use a high resolution only near sharp transition regions and singularities but a moderate resolution in regions with smooth, slowly varying behavior of the solution.

For this purpose, numerical schemes have been discussed or are under current investigation that aim at adapting the *spatial* grid to the local behavior of

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