ON THE STABILITY OF FULLY ADAPTIVE MULTISCALE SCHEMES FOR CONSERVATION LAWS USING APPROXIMATE FLUX AND SOURCE RECONSTRUCTION STRATEGIES*

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Abstract. In order to accelerate finite volume schemes applied to (inhomogeneous) hyperbolic conservation laws multiresolution based adaptive concepts can be used. The basic idea is to analyze the local regularity by means of a multiresolution analysis of cell averages. By difference information between successive refinement levels local grid adaptation is triggered employing threshold techniques. This leads to a significant gain in computational complexity. The crux is to compute numerical fluxes and sources on local resolution levels such that the overall accuracy of the reference solution on the finest discretization is maintained. In the present work a modified approach based on polynomial reconstruction techniques is introduced and investigated analytically. The efficiency and accuracy of the adaptive concept is significantly improved, in particular for inhomogeneous equations. This is confirmed by numerical parameter studies.

Key words. conservation laws, finite volume schemes, grid adaptation, biorthogonal wavelets

AMS subject classifications. 35L65, 65M12, 65M50, 65T60, 74S10

1. Introduction. Nowadays finite volume methods are routinely used for the discretization of conservation laws as they arise, for instance, in computational fluid dynamics. Here, due to the inhomogeneity of the solutions, adaptive grid methods can significantly improve the efficiency by concentrating cells only where they are most required, while reducing storage requirements as well as the computational time.

For this purpose, numerical schemes have been discussed or are under current investigation that aim at adapting the *spatial* grid to the local behavior of the flow field. In the early 90's Harten [13] proposed to use *multiresolution techniques*. The cell averages on a given highest level of resolution (*reference mesh*) are represented as cell averages on some coarse level where the fine scale information is encoded in arrays of *detail coefficients* of ascending resolution that reveals insight into the local behavior of the solution. This multiresolution framework has been extended to multidimensional problems [3, 1, 10, 8, 20] on Cartesian, curvilinear and unstructured meshes, respectively.

In Harten's original approach the multiresolution analysis is used to control a hybrid flux computation by which computational time for the flux computation can be saved whereas the overall computational complexity is not reduced but still stays proportional to the number of cells on the uniformly fine reference mesh. Opposite to this strategy, threshold techniques are applied to the multiresolution decomposition in [12, 18, 9, 16, 21] where detail coefficients below a threshold value are discarded. By means of the remaining significant details a locally refined mesh is determined whose complexity is significantly reduced in comparison to the underlying reference mesh. A comparison of Harten's original framework and the fully adaptive framework can be found in [6].

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