ADAPTIVE TIMESTEP CONTROL FOR INSTATIONARY SOLUTIONS OF THE EULER EQUATIONS*

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Abstract. In this paper we continue our work on adaptive timestep control for weakly instationary problems [29, 30]. The core of the method is a space-time splitting of adjoint error representations for target functionals due to Süli [32] and Hartmann [18]. The main new ingredients are (i) the extension from scalar, 1D, conservation laws to the 2D Euler equations of gas dynamics, (ii) the derivation of boundary conditions for a new formulation of the adjoint problem and (iii) the coupling of the adaptive time-stepping with spatial adaptation. For the spatial adaptation, we use a multiscale-based strategy developed by Müller [24], and we combine this with an implicit time discretization. The combined space-time adaptive method provides an efficient choice of timesteps for implicit computations of weakly instationary flows. The timestep will be very large in regions of stationary flow, and becomes small when a perturbation enters the flow field. The efficiency of the solver is investigated by means of an unsteady inviscid 2D flow over a bump.

Key words. compressible Euler equations, weakly instationary flows, adaptive time-stepping, adjoint error analysis, multiscale analysis.

AMS subject classifications. 35L65, 76N15, 65M12, 65M15, 65M50.

1. Introduction.

Today, there is broad consensus that the numerical solution of compressible flow equations requires a highly resolved mesh to simulate accurately the different scales of the flow field and its boundaries. Adaptive grid methods can significantly improve the efficiency by concentrating cells only where they are most required, thus reducing storage requirements as well as the computational time. There has been a tremendous amount of research designing, analyzing and implementing codes which are adaptive in space, see e.g. [6, 24, 22, 25] and references therein.

Here our interest is in timestep adaptation. For stationary problems, local timesteps which are linked to the spatial gridsize are commonplace, and they are heavily built upon the fact that time-accuracy, or time synchronization is not needed. On the other hand, for fully instationary flows, explicit algorithms whose timestep is governed by the *CFL* restriction of at most unity are the method of choice. In [30], we began to explore one of the remaining gaps, namely weakly instationary flows on which we will focus in the following. Many real world applications, like transonic flight, are perturbations of stationary flows. While time accuracy is still needed to study phenomena like aero-elastic interactions, large timesteps may be possible when the perturbations have passed. For explicit calculations of instationary solutions to hyperbolic conservation laws, the timestep is dictated by the *CFL* condition due to Courant, Friedrichs and Lewy [9], which requires that the numerical speed of propagation should be at least as large as the physical one. For implicit schemes, the *CFL* condition does not provide a restriction, since the numerical speed of propagation is infinite. Depending on the equations and the scheme, restrictions may come in via the stiffness of the resulting nonlinear problem. These restrictions are usually not as strict as in the explicit case, where the *CFL* number should be below unity. For implicit calculations, *CFL* numbers of much larger than 1 may well be possible. Therefore, it is a serious question how large the timestep, i.e. the *CFL* number, should be chosen.

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