On the Existence of the Composite Curve near a Degeneration Point

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Abstract

The paper is concerned with the analysis of the composite curve. This is a special type of curve in phase space which arises in the construction of the solution of a Riemann problem for a system of strictly hyperbolic conservation laws which exhibits non–genuinely nonlinear characteristic fields. It will be shown that this curve uniquely exists near a degeneration point of the corresponding characteristic field. Furthermore it is tangentially connected to the admissible part of the rarefaction curve breaking off at this point. The proof is based on Liapunov-Schmidt Reduction which reduces the system to an equivalent scalar equation. Then this equation is solved by means of an ansatz common in bifurcation theory.

Key Words: Riemann problem, composite curve, Liapunov–Schmidt reduction.

AMS Subject Classification: 35L65, 35L67, 35Q05, 70H35, 76N10

1 Introduction

We are concerned with the Riemann problem for a system of conservation laws in 1D

$$u_t + f(u)_x = 0 \tag{1}$$

and piecewise constant initial data

$$u(0,x) = \begin{cases} u_l & , \quad x < 0 \\ u_r & , \quad x > 0 \end{cases} \tag{2}$$

for two states $u_l,\ u_r$ in the admissible phase space $\mathcal{D} \subset \mathbb{R}^n$. Here $u : \mathbb{R}_+ \times \mathbb{R} \to \mathcal{D}$ denotes the vector of n conservative quantities and $f : \mathcal{D} \to \mathbb{R}^n$ is the flux vector which is supposed to be sufficiently smooth, i.e., $f \in C^3(\mathcal{D})$.

The Riemann problem has been subject of extensive research. Significant contributions concerning the construction of its solution have been made by Lax [Lax57], Gelfand [Gel59] and Liu [Liu75]. In the sequel, we consider the general frame as in [Liu75]. For this purpose, we assume that the system of conservation laws (1) is *strictly hyperbolic*, i.e., there is a complete set of eigenvalues $\lambda_k(u),\ k = 1, \ldots, n$, of the Jacobian of f such that

$$\lambda_1(u) < \ldots < \lambda_n(u) \qquad \forall\, u \in \mathcal{D}.$$

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