H-Adaptive Multiscale Schemes for the Compressible Navier-Stokes Equations – Polyhedral Discretization, Data Compression and Mesh Generation

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Summary

In this paper we present the main conceptual ingredients and the current state of development of the new solver QUADFLOW for large scale simulations of compressible fluid flow and fluid–structure interaction. In order to keep the size of the discrete problems at every stage as small as possible for any given target accuracy, we employ a multiresolution adaptation strategy that will be described in the first part of the paper. In the second part we outline a new mesh generation concept that is to support the adaptive concepts as well as possible. A key idea is to understand meshes as parametric mappings determined by possibly few control points as opposed to store each mesh cell separately. Finally, we present a finite volume discretization along with suitable data structures which again is to support the adaptation concepts. We conclude with numerical examples of realistic applications demonstrating different features of the solver.

1 Introduction

The numerical simulation of non–stationary fluid–structure interactions poses enormous challenges to computing resources, data management strategies but also to the underlying mathematical concepts that contribute to keeping the computational complexity tractable. However, any increase in computing power is bound to be surpassed by the demands put forth by scientist and engineers through more and more realistic and consequently more complex models. Advancing the current frontiers in numerical simulation calls for combined efforts beyond traditional borders of scientific disciplines. Studying the interaction of aerodynamics and structural mechanics in complex geometries is a typical example. The fact that several severe obstructions such as time–dependency of the involved processes, varying complex geometries and the coupling of physical regimes with different characteristic features combined with the huge amount of data needed to model the process in an

adequate way, indicates the principal limitations of conventional approaches. In order to resolve a typically singular behavior of the solution in complex geometries meshes with several millions of cells are required. Improved hardware or purely data oriented strategies such as parallel computing are not sufficient to overcome the arising difficulties. As important and necessary these aspects may be they have to be complemented in the long run by mathematical concepts that aim at minimizing in the first place the complexity and size of arising discrete problems.

This paper summarizes some recent attempts in this direction. It describes the present stage of an integrated development of dynamic adaptation strategies, mesh generation and discretization which is presented here for a still intermediate state of two–dimensional gas flow. The central objective is to realize adaptively generated discretizations that are able to resolve the physically relevant phenomena at the expense of possibly few degrees of freedom and correspondingly reduced storage demands. This requires a careful coordination of the core ingredients namely the discretization of the underlying system of partial differential equations, the generation and management of suitable meshes and the adaptation mechanisms, all three parts being based on suitable data structures. The following remarks are to indicate the main orientation. Accepting the Navier Stokes equations as the model of choice we give preference to quadrilateral and hexahedral meshes that still facilitate best boundary fitted anisotropic meshes. To retain sufficient geometric flexibility this is combined with block structuring. Complex geometry as well as the anticipated fine scale behavior of the solutions require extremely high resolution at least in parts of the computational domain.

To avoid storage demands which, in the light of the envisaged applications are prohibitive, we wish to employ local mesh refinement. A pivotal role is therefore played by reliable and efficient refinement strategies that are presented in Section 3.1. The main distinction from previous work in this regard lies in the fact that we employ here recent *multiresolution techniques*. The starting point is a proposal by Harten to transform the arrays of cell averages associated with any given finite volume discretization into a different format that reveals insight into the local behavior of the solution. The cell averages on a given highest level of resolution are represented as cell averages on some coarse level where the fine scale information is encoded in arrays of *detail coefficients* of ascending resolution. This requires a *hierarchy of meshes*. In contrast to Harten's idea the multiscale representation is not only used to avoid expensive flux evaluations in regions where the solution is smooth but to create *locally refined meshes*. As long as one works on a uniform mesh (even when in a major part inexpensive finite differences are employed) the computational complexity stays proportional to the number of cells which in 3d calculations with the above objectives is prohibitive. Thus a principal objective is to extract the inherent complexity of the problem by placing as few degrees of freedom so as to still capture the features of the searched for solution within a given tolerance. A central mathematical problem is then to show that the essential information to be propagated in time is still kept with sufficient accuracy when working on locally coarser meshes.

The adaptation strategy gives rise to locally refined meshes of quadtree respectively octree type. The second important ingredient is the generation of such meshes along with the information needed by the flow solver at any stage of a dynamical calculation. A key idea is to represent such meshes with as few parameters as possible while further successive refinements can be efficiently computed based on the knowledge of these parameters. This seems to be of vital importance with regard to (geometrically) non–stationary processes. Roughly speaking the mesh in each block results from evaluating a parametric mapping from the computational domain into the physical domain. Such mappings can be based on B-spline representations in combination with well established concepts from CAGD (computer aided geometric design). The quantities to be updated in time are the relatively few control parameters in those parametric representations, while mesh points on any level of resolution can be efficiently computed due to the locality of the B-spline representation. The fact that one needs indeed only relatively few control points in order to generate meshes of good quality is partly due to the variation diminishing property of B-splines. For this statement we refer to the discussion later in Section 4 of this paper.

Finally, Section 5 is devoted to the outline of a discretization scheme that meets the requirements of the adaptation concept and fits well with the mesh generation. To avoid complicated mesh management within each block and to keep the discretizations of the individual blocks as independent as possible and, in particular, to avoid global geometrical constraints, we insist on meshes with hanging nodes. Since the discretization is based on the finite volume concept this appeared to us as the best compromise. This requires the development of a finite volume scheme for fairly general cell partitions that can cope, in particular, with hanging nodes and possible unstructured parts in complicated regions of the flow domain. Section 5 offers a self contained account of the discretization scheme including data structures, realization of spatial second order, the treatment of convective and viscous fluxes, choice of limiters, the treatment of boundary conditions and the validation of the basic scheme by a proper selection of test cases.

We conclude with several applications to well–known numerical and fluid dynamical test cases that highlight the features of the whole flow solver.