The influence of a source term, an example: chemically reacting hypersonic flow

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Abstract

We show, by way of an example, that the solution of a system of hyperbolic conservation laws exhibits an unexpected behavior if a source term is present. The example is the system of Euler equations for N species in two space dimensions. If the source term is not present and in the initial and inflow conditions a fixed mixture of species is prescribed then the solution basically behaves like the flow of an ideal gas, except that there are additional equations for the different species. However, introducing the chemical reaction terms produces a thin boundary layer, which makes numerical computations of the two-dimensional problem extremely difficult, if not impossible for todays computers. In addition this boundary layer is unphysical. We shall analyse the boundary layer in a one-dimensional calculation along the stagnation point stream line. In [11] a modification of the Van Leer flux vector splitting is presented which is able to indicate the presence of the boundary layer in a two-dimensional calculation.

1 Introduction

When developing numerical schemes for hyperbolic conservation laws, most authors consider a scalar one-dimensional conservation laws without a source term. New phenomena have to be taken into account when one wants to generalize such methods to systems in several space dimensions. Here we consider an effect which arises from an interaction of the source term with a geometric singularity. We consider hypersonic Euler flow of N chemically reacting species around a blunt body in two space dimensions. We shall indicate that an extreme chemical boundary layer will develop at the stagnation point. However, if the chemical reactions are turned off then no boundary layer develops. In Section 2 we formulate the mathematical problem and report on the fact that todays 'general purpose' numerical schemes with todays computers do not give the correct stagnation point temperature. In Section 3 we consider analytically the steady state flow along the stagnation point stream line if one assumes that the flow is symmetric with respect to this line. This leads to a free boundary value problem for a system of $N+2$ ordinary differential equations. If one uses N equations for the mass fractions and one for the pressure and velocity along the stagnation point stream line, one can observe nicely the effect of the source term. A heuristic argument is given to show why the boundary layer depends on the geometry of the body and the reaction rates. In Section 4 we give a brief outline of a scheme which is able to at least indicate the existence of a boundary layer in two-dimensional computations.

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