An adaptive staggered grid scheme for conservation laws

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We present an adaptive staggered grid scheme in two spatial dimensions for the approximate solution of hyperbolic systems of conservation laws. It is based on the second order central scheme of Nessyahu and Tadmor [7] and its extension to two-dimensional cartesian grids by Jiang and Tadmor [3]. The ease of evaluating fluxes not on the border but inside a cell and the componentwise application of the scalar framework to solve systems of conservation laws make this scheme very convenient to work with. No (approximate) Riemann solvers, field-by-field decompositions, etc. are required. The resulting algorithms are particularly simple and computationally very efficient. Moreover, they can be easily applied to problems where no Riemann solvers exist (see for example [8] for an application to granular avalanches).

Besides these obvious advantages of the central framework, there are also algorithmical difficulties caused by the use of staggered grids. The most well-known difficulty is the smearing of contact discontinuities. Another problem is presented by grid-orientation effects which occur for radially symmetric flows. Both of these issues are treated in the forthcoming paper [6]. There are other difficulties which are more closely related to the grid structure itself rather than the particular solver used on the grids. One of these issues is the treatment of boundaries of the computational domain, see e.g. [5]. Another challenging task, which is particularly important for practical applications, is local adaptive grid-refinement and coarsening, in particular for unsteady flows. Since the grids are now staggered, new techniques need to be developed. These technical difficulties have prompted several authors to leave the staggered grid approach by projecting the intermediate solution back onto the original grid, see for example [2, 4]. Here we attack the problem of local grid-adaptation *directly* using structured staggered grids. We would like to mention that *unstructured* adaptive staggered grids have been developed earlier by Arminjon, Viallon and co-workers, and were sucessfully applied to steady flows (see [1] and the references therein).

Given an adaptively refined rectangular grid (the *original grid*) there is no unique choice of a corresponding staggered *dual grid*. Our approach relies on the following natural design principles:

1. the *corners* of the staggered cells should lie in the *interior* of the original cells, and vice versa (this is our definition of staggered grids)

2. the local resolution of the dual grid should reflect that of the original grid

3. the edges of the dual cells should be parallel to the axis, but we do *not* require the dual cells to be rectangular.

As our original grid we use an adaptive rectangular grid organized in a quadtree. We construct the staggered dual grid locally on each cell $\mathcal{C}$ of the original grid following simple rules considering only the size of the direct neighbours of $\mathcal{C}$. Adding a few auxiliary nodes to the list of nodes of the original grid in exceptional situations, we can guarantee that each cell of the dual grid contains exactly one node of the original grid. Thus we can handle the dual grid by storing the nodes of the original grid in a hash-table.

As the staggered grid corresponding to the dual grid we use the original grid again. We refine and coarsen only the original and not the dual grid.

Figure 1 below shows an example of the two corresponding grids, the original one with solid lines and the dual grid with broken lines.

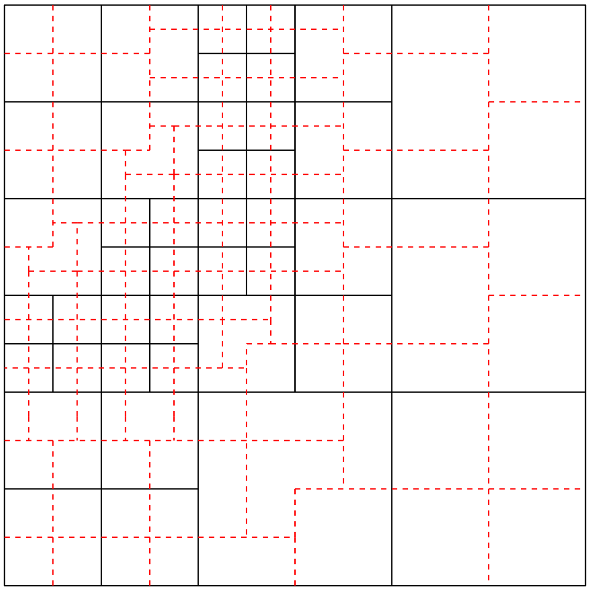

Figure 1: original and corresponding dual grid

We have applied our scheme to various one- and two-dimensional test problems, e.g. the rotating cone, Sod's and Lax' shock tube and the forward facing step for the Euler equations of gas dynamics, as well as the Brio-Wu Riemann problem and the Ország-Tang vortex for the equations of ideal magnetohydrodynamics. In Figure 2 we display numerical results for the forward facing step, using up to nine levels of local grid-refinement, which corresponds to 512 cells in the x-direction on the finest grid. Note that for this problem, special care has to be taken at the boundary, where the first of our design principles cannot be enforced. Details will be presented in the proceedings.

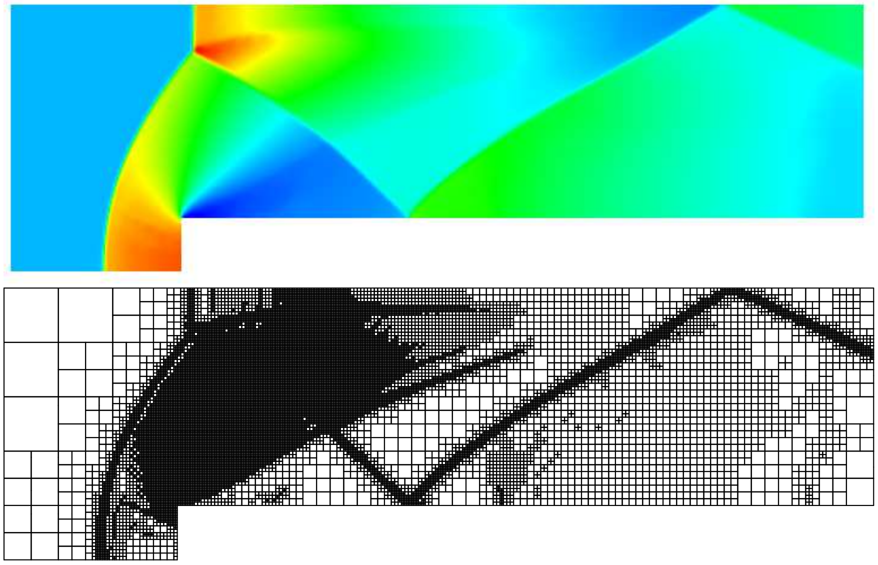

Figure 2: forward facing step

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This work was supported by DFG-SPP ANumE.