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Michael Rom, Karl-Heinz Brakhage, Sebastian Barth, Christian Wrobel, Patrick Mattfeld and Fritz Klocke

Institut für Geometrie und Praktische Mathematik
Templergraben 55, 52062 Aachen, Germany

Michael Rom, Karl-Heinz Brakhage
Institute for Geometry and Applied Mathematics (IGPM), RWTH Aachen, Templergraben 55, 52056 Aachen, Germany
{rom,brakhage}@igpm.rwth-aachen.de
Sebastian Barth, Christian Wrobel, Patrick Mattfeld, Fritz Klocke
Laboratory for Machine Tools and Production Engineering (WZL), RWTH Aachen, Steinbachstr. 19, 52074 Aachen, Germany
{s.barth,c.wrobel,p.mattfeld,f.klocke}@wzl.rwth-aachen.de

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Michael Rom, Karl-Heinz Brakhage
Institute for Geometry and Applied Mathematics (IGPM),
RWTH Aachen, Templergraben 55, 52056 Aachen, Germany
{*rom,brakhage*}*@igpm.rwth-aachen.de*
Sebastian Barth, Christian Wrobel, Patrick Mattfeld, Fritz Klocke
Laboratory for Machine Tools and Production Engineering (WZL),
RWTH Aachen, Steinbachstr. 19, 52074 Aachen, Germany
{*s.barth,c.wrobel,p.mattfeld,f.klocke*}*@wzl.rwth-aachen.de*

Abstract

The proper choice of a grinding tool is essential for a productive grinding process and a high quality of the resulting work piece surface. Hence, the grinding wheel structure consisting of abrasive grain material, bonding material and pores has to be composed wisely. We present a new approach for mathematically modeling such grinding wheel structures with the objective of predicting the volumetric composition of the grinding wheel components such that grinding requirements can be met without using trial and error methods. For the model, we focus on a small element of a grinding wheel, such as a cube, which we call volumetric structure element (VSE). In this paper, we concentrate on several aspects of the modeling procedure, namely the initial grain arrangement, a collision-free grain rotation and translation algorithm to obtain required grain volume fractions in a VSE, the tetrahedral mesh generation for a whole VSE and the modeling of ceramic bond.

Keywords: Mathematical Grinding Wheel Modeling, Granular Structures, Ceramic Bond, CBN

1 Introduction

Grinding wheels are complex high precision tools applied to a great variety of work piece materials. They are used for finishing operations to ensure a high surface quality as well as for high material removal processes. The major components are abrasive grain material, bonding material and pores, where grain diameters vary between a submicrometer range and about $500\,\mu m$. The main conventional abrasive grain materials are corundum and silicon carbide, whereas cubic boron nitride (CBN) and diamond with a higher degree of hardness are the so-called superabrasive materials. The main advantages of CBN in comparison with diamond are its thermal stability and the applicability to grinding ferrous materials [1]. Hence, we concentrate on CBN as grain material. The grains are bonded using vitrified (ceramic),

resin (polymer) or metallic bonding systems. Extensive information on grinding can be found in [2].

Most previous works for modeling and simulation of grinding processes focus on the grinding wheel topography, i.e., on the surface of the grinding wheel, see [3, 4, 5]. The modeled topography is often used as an input parameter for the development of force models or for the simulation of abrasive wear. In a more recent work, Schumann et al. [6] go one step further: they have developed a model in which they distribute grains with a CBN-like shape randomly on the grinding wheel surface. In the simulation, the grains can be intersected during the cutting process to reflect the topography. However, the appropriate composition of the grinding wheel structure influences the topography significantly. Hence, the composition is essential for the grinding result, i.e., for the final work piece surface quality. Since determining a suitable composition is a process nearly exclusively based on experience, it can be very expensive and time-consuming. We develop an approach for mathematically modeling the grinding wheel structure with the objective of predicting the volumetric composition of the grinding wheel components such that grinding requirements can be met without using trial and error methods. Basic concepts and preliminary algorithms for this purpose were published in [7]. In this paper, we concentrate on the modeling of ceramic-bonded grinding wheel structures.

For our model, we focus on a small volumetric structure element (VSE) of a grinding wheel. The grains can be chosen from convex polyhedra representing CBN-like shapes or from scan data obtained from computer tomography (CT). After initially arranging the grains according to a close-packing of spheres, they can be moved in the VSE to obtain the required grain volume fraction prescribed by the particular grinding wheel specification. For this purpose, we have developed a collision-free grain rotation and translation algorithm. Ceramic bond bridges between grains are modeled by applying an approach combining analytical and discrete calculations, implemented as an iterative algorithm to make sure to meet given bond volume fractions.

2 Mathematical Model

In the following, we describe our mathematical modeling approach to obtain a VSE. We discuss the choice of the grain shapes, the initial arrangement of the grains, the compaction of grains to comply with grain volume fractions given by grinding wheel specifications, the tetrahedral meshing of a VSE and the modeling of ceramic bond.

2.1 Grain Modeling

The VSE can be composed of grains extracted from CT scans. The procedure for the generation of triangle meshes for the scanned grains is described in [7]. Usually, such a grain mesh has a high number of vertices, edges and faces. Since the grains later have to be rotated and translated in space to obtain a VSE with a given

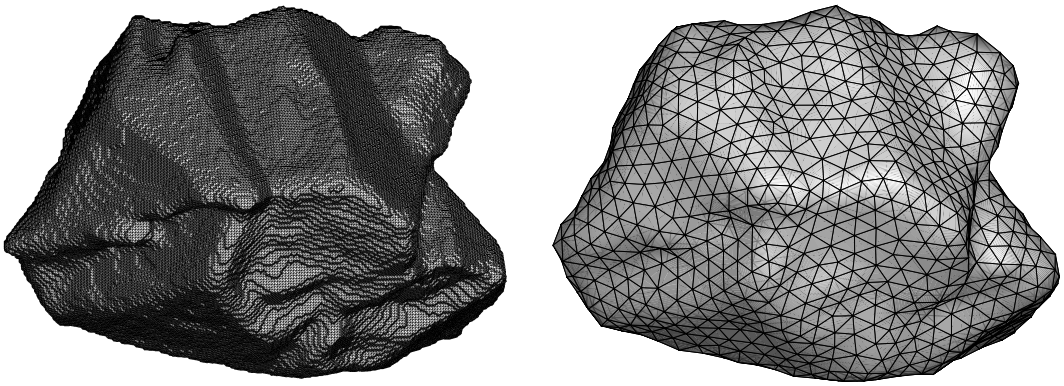


Figure 1: Original mesh for a grain from a CT scan with $\approx 198,000$ triangles (left) and after mesh decimation with $\approx 2,200$ triangles (right).

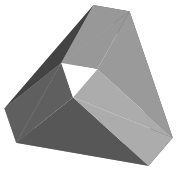
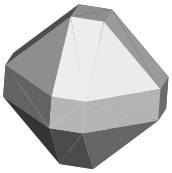
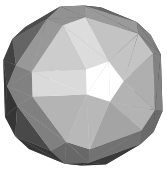
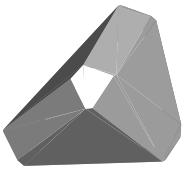
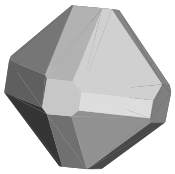
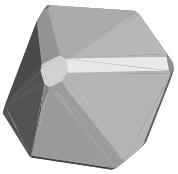
	tetrahedron	octahedron	hexahedron	icosahedron
cut vertices/edges				
randomized				

Table 1: Convex model grain shapes based on platonic solids.

grain volume fraction, it is essential to decimate the grain meshes to reduce the computation time of geometric applications such as collision detection. Hence, we use a mesh decimation algorithm by Kobbelt et al. [8] which maintains the topology of the grains within a prescribed tolerance. An example for an original grain mesh and its decimated counterpart is depicted in Fig. 1.

For the development of a mathematical structure model and appropriate algorithms, we first consider simpler geometries for the grains, precisely convex polyhedra based on platonic solids. We have incorporated tetrahedra, octahedra, hexahedra and icosahedra which can be modified by randomly changing angles and edge lengths and by randomly choosing planes to cut off corners or edges of the polyhedra, see Tab. 1, to obtain grain shapes similar to CBN. For the generation of triangle meshes for these grains, we use the convex hull algorithm *Quickhull* [9].

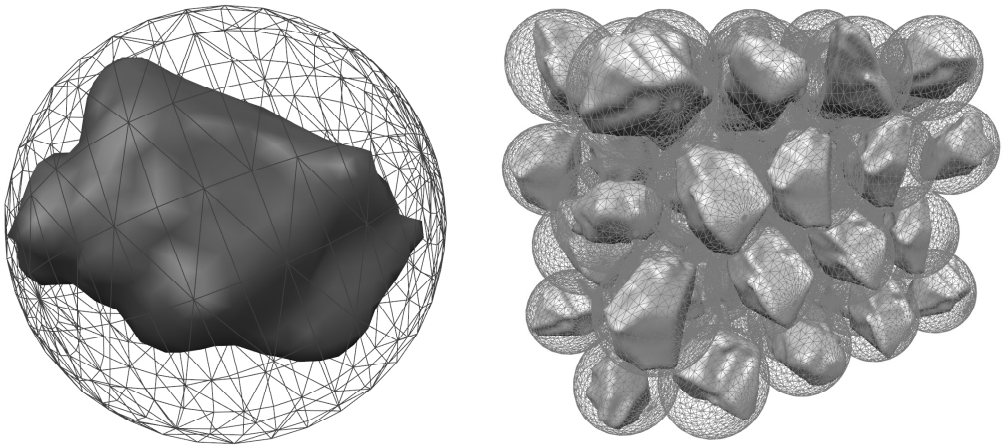


Figure 2: Smallest bounding sphere for a single grain (left) and hexagonal close-packing of equal spheres (right).

2.2 Initial Grain Arrangement

For the initial grain arrangement in our mathematical structure model, we first compute the smallest bounding sphere for each grain by applying an algorithm published by Gärtner [10]. The grains are distributed such that a hexagonal close-packing of equal spheres is obtained where the maximum bounding sphere radius is used for all grains. Examples are shown in Fig. 2. The VSE is created by cutting out the contents of a chosen cuboidal bounding box.

It is well-known that a hexagonal close-packing of equal spheres can fill up space with a density of $\frac{\pi}{3\sqrt{2}} \approx 74\,\%$ [11]. Considering this information, we can increase the bounding sphere radii if the grain volume fraction in the VSE is larger than required by an investigated grinding wheel specification. Hence, the target value can be matched exactly. On the other hand, we have to compact the grains if the grain volume fraction is too low. This is discussed next.

2.3 Compaction of Grains

The compaction of the grains in a chosen bounding box representing the VSE has to be done collision-free. We have developed an algorithm which moves the grains into the direction of the center point of the bounding box or to one of the three coordinate planes through the center point. The moving direction can be randomized by allowing a deviation from the original direction. Then, the maximum possible moving length without collision with another grain is computed. The algorithm passes through a list of the grains which is sorted depending on the distance of the grains to the center point/plane, i.e., the first translation is calculated for the grain closest to the center point/plane.

In addition to translation, it is also possible to rotate the grains before moving

them. For this purpose, bounding ellipsoids of the grains are computed by using a variant of Khachiyan's algorithm [12]. The grains are temporarily rotated around the principal axes of their ellipsoid to find the case with the best possible moving length which determines the actual rotation. The algorithm for the case with the bounding box center point as the target point is summarized in Alg. 1.

Algorithm 1: Grain compaction

```
Sort the grains depending on their distance to the VSE center point;
for each grain from the sorted list do
    Compute the moving direction;
    Compute the bounding ellipsoid;
    for each principal axis do
        for a user-given number n of iterations do
            Compute the rotation angle depending on n;
            Rotate the grain around the current principal axis;
            Check for collisions with other grains;
            if there is no collision then
                Compute the maximum possible moving length;
                if the max. poss. moving length is the best so far then
                    Store the current rotation angle, axis and maximum
                    possible moving length as best setting;
                end
            end
        end
    end
    Rotate and translate the grain corresponding to the best setting;
end
```

Details on the computation of possible collisions can be found in [7]. For an efficient collision detection, a preselection of necessary intersection tests is done, e.g., by sorting out mesh faces which lie on the back side of a grain regarding the moving direction such that they cannot be intersected when translating the grain. Furthermore, the number of intersection calculations is reduced considerably by applying simple and fast tests such as the detection of bounding sphere intersections first.

An example for a VSE containing 125 grains generated as randomized tetrahedra is illustrated in Fig. 3. In the initial arrangement, the depicted bounding box contains a grain volume fraction of about 16.1 %. After five iterations by only translating into the direction of the center point (without any randomized deviation from that direction), the grain volume fraction is increased to 38.7 %. The increase from the fourth to the fifth step is only 0.1 percentage point. However, one additional iteration in which also rotations in steps of five degrees are applied leads to a further

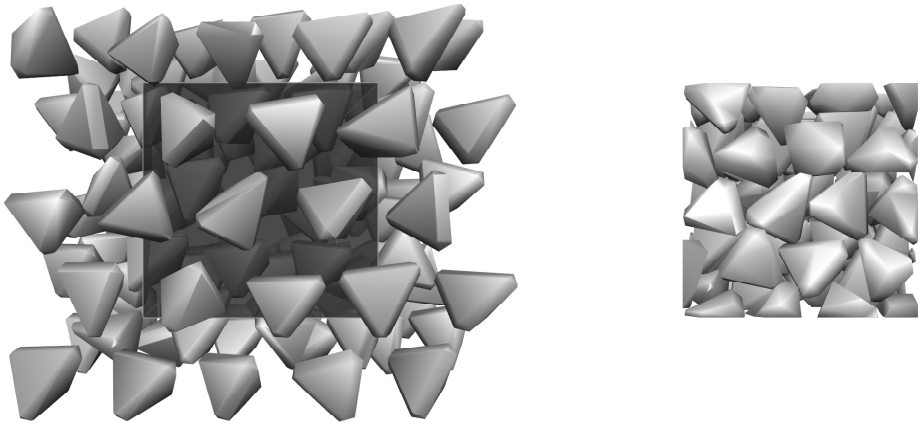


Figure 3: Initial grain arrangement with target bounding box (left) and VSE after six compaction iterations (right).

significant increase to 48.8 %. With the parameters applied in this example, a maximum grain volume fraction of about 54 % is attainable by continuing the iterative process.

When the VSE is created, the grains have to be cut with the bounding box as visible on the right-hand side of Fig. 3. Hence, the resulting polygons on the six bounding planes of the box have to be remeshed. In the case of convex grains, the above-mentioned *Quickhull* algorithm [9] can be used for this purpose. In the case of non-convex grains from CT scan data, the polygons on the bounding planes can also be non-convex and they can contain holes. Then, we apply the mesh generator *Triangle* by Shewchuk [13].

2.4 Tetrahedral Mesh Generation

Once a VSE has been created, we can apply the tetrahedral mesh generator *TetGen* [14] to mesh the interior of the domain. As a constraint, the maximum tetrahedral cell volume can be prescribed. In the future, we want to use the tetrahedral meshes for finite element simulations to predict the change of the grinding wheel topography, e.g., by grain break-out or bond fracturing, depending on material properties. Currently, we make use of the tetrahedral meshes for the generation of ceramic bond as explained in the next section.

2.5 Ceramic Bond Modeling

Ceramic-bonded grinding wheels are produced by a sintering process. Grains and bonding material in powder form are mixed and pressed into a mold. The bonding material is liquefied under high heat such that it flows around the grains. When the bonding material solidifies after the heating, bond bridges are formed between

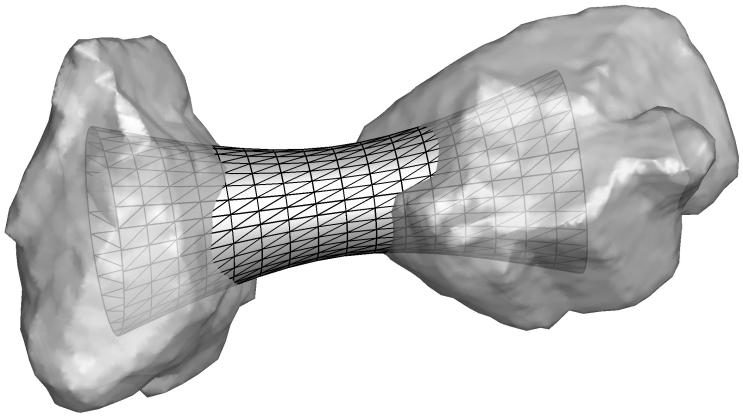


Figure 4: Grains with bond bridge modeled by surface mesh (translucent view).

the grains. Due to the minimization of the surface tension, these bond bridges correspond to minimal surfaces, cf. [15, 16].

We model the ceramic bond by approximating the minimal surfaces with splines. First, we determine to which adjacent grains each grain will be connected by bond depending on the distance between the grain centers. For each of the two grains of a bonded connection, we compute the largest possible circle which is perpendicular to the axis through the two grain centers and which lies completely inside the particular grain. A third circle with a smaller radius is defined in the middle of the segment which connects the two other circle centers. After that, spline curves are calculated, each incorporating one point on each of the three circles and points in between. By meshing the surface enclosed by the spline curves, we end up with bond surface meshes such as the one shown in Fig. 4 (with the grains in a translucent view).

Since the bond meshes intersect the grain meshes, we only use them to determine the tetrahedral volume cells which define a bond bridge. This is done by finding intersections between tetrahedra and the bond surface meshes. The result for the example is presented in Fig. 5.

For a more realistic rounded connection of the bond bridges to the surfaces of the grains, we grow the bond bridges at the grains by introducing further layers of tetrahedral bond cells. One layer contains potential new bond cells which are tetrahedral cells being adjacent to bond cells and connected to grain triangle faces. Depending on the angle between the bond bridge axis and the normal vector of the grain face, a potential bond cell is introduced as new bond cell or not. If the angle is zero, the bond bridge axis is perpendicular to the triangle face. Hence, it is reasonable to add more layers of bond cells for small angles than for large angles where a rounded connection of a bond bridge to a grain already exists. The result for the example of Fig. 5 after three bond growth iterations for maximum angles of 30°, 60° and 90°, respectively, is illustrated in Fig. 6.

By iteratively adapting the radii of the circles in the middle of each bond bridge,

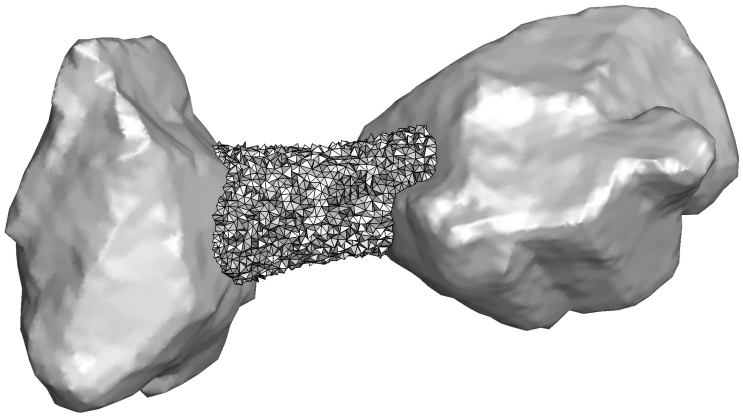


Figure 5: Grains with bond bridge modeled by tetrahedral cells.

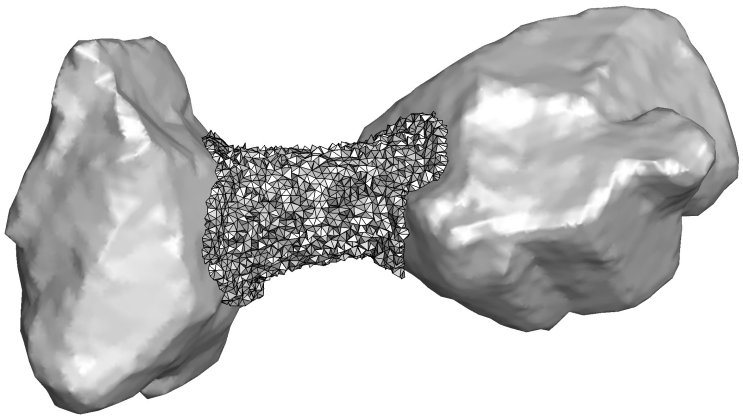


Figure 6: Grains with bond bridge modeled by tetrahedral cells after bond growth at the grain surfaces.

we make sure that a prescribed bond volume fraction for the whole VSE is met. The algorithm for the ceramic bond modeling is summarized in Alg. 2.

A part of a structure cut of a specimen, recorded by light-optical microscopy, can be seen in Fig. 7. The original picture [17] shows an area of $4\,mm \times 4\,mm$ from which we have taken a part with a size of approximately $0.8\,mm \times 0.8\,mm$ here. The specimen contains CBN grains with a mean grain size of $91\,\mu m$. It has a grain volume fraction of 40 % and a bond volume fraction of 20 %. A slice through a modeled VSE with randomized tetrahedra generated for the same parameters is depicted in Fig. 8. Both pictures reflect the dense packing of the grains and the bond bridges.

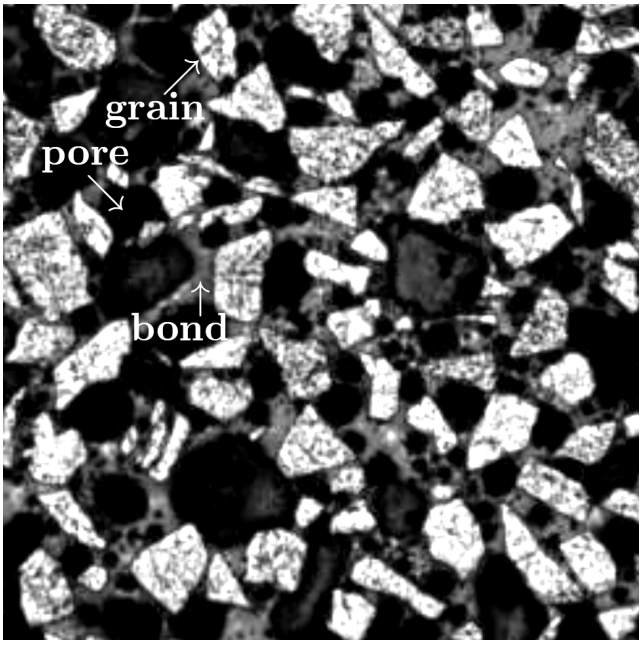


Figure 7: Structure cut of a specimen with CBN grains and ceramic bond.

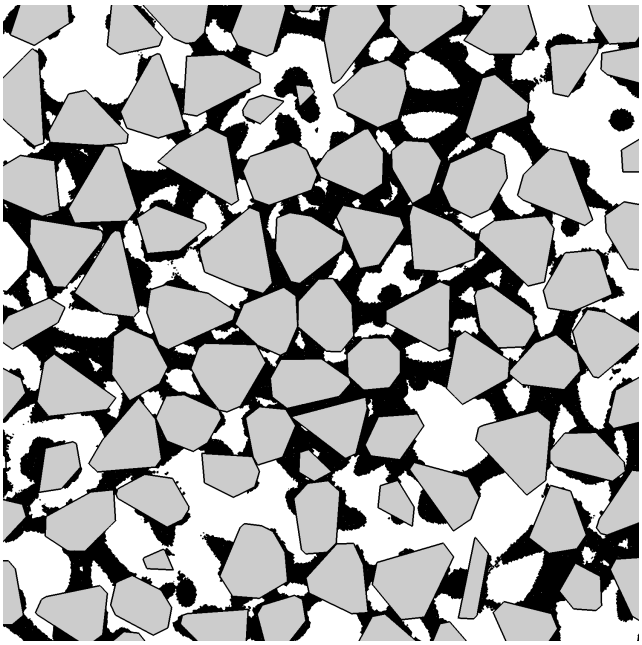


Figure 8: Slice through a VSE with randomized tetrahedra and the same grain and bond volume fractions as in Fig. 7 (gray: grains, black: bond).

Algorithm 2: Ceramic bond modeling

```
for each grain do
    Find the adjacent grains which are to be connected by bond bridges;
end
for each bond bridge between two grains do
    Compute the axis through the two grain centers;
    For both grains, compute the maximum circle perpendicular to the axis
    and completely inside the particular grain;
    Compute a circle in the middle of the bond bridge with a radius of one
    half of the minimum of the other two radii;
end
repeat
    for each bond bridge between two grains do
        Compute spline curves each incorporating one point on each of the
        three circles;
        Triangulate the surface induced by the spline curves;
        Mark all tetrahedral cells whose center point lies "inside" the bond
        surface mesh as bond cells;
        Grow the bond bridge at the grain surfaces by introducing layers of
        new bond cells;
    end
    if the overall bond volume fraction is too low then
        Increase the middle circle radii;
    else
        Decrease the middle circle radii;
    end
until the overall bond volume fraction differs from the prescribed value by less
than a tolerance;
```

3 Conclusion and Future Work

In this paper, we have presented an approach for modeling ceramic-bonded grinding wheel structures. Our model incorporates non-convex grains from CT scan data as well as simplified convex grains. The grains are triangulated and the resulting meshes are decimated if necessary. The initial step for modeling a volumetric element of a grinding wheel is the computation of the smallest bounding sphere for each grain and the subsequent arrangement of the grains according to a hexagonal close-packing of equal spheres. To obtain larger grain volume fractions, the grains in our model can be compacted using our collision-free rotation and translation algorithm. After a tetrahedralization of the VSE, ceramic bond bridges between the grains can be modeled by computing approximated minimal surfaces and by finding all tetrahedral

volume cells which are enclosed by these surfaces.

In future investigations, we want to take into account grain fracturing, grain break-out and bond fracturing for an appropriate modeling of the grinding wheel topography. This can be done by considering statistical distribution functions determined by experiments or by modeling material properties. A verification of our model should then be done for different grinding wheel specifications.

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