On a Robust Adaptive Multigrid Solver for Convection–Dominated Problems

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April 21, 1999

Abstract

The objective of this paper is to develop and describe an adaptive multigrid scheme for the numerical solution of convection-diffusion-reaction equations with dominating convection or reaction term. Specifically, we propose a systematic construction of problem and *scale dependent* restriction, prolongation and smoothing operators in combination with an *adaptive mesh refinement* strategy. In particular, adaptivity is not only to reduce computational complexity but also to *stabilize* standard Galerkin discretizations so that adding artificial viscosity can be avoided. The approach is suited for any choice of underlying multiresolution sequences, in particular, for standard finite element discretizations on nested but possibly nonuniform meshes. Moreover, all concepts are in principle independent of the spatial dimension and avoid any special flow dependent enumeration of unknowns. The performance of the scheme is illustrated by various examples in one and two spatial dimensions. They are chosen so as to exhibit possibly many types of adverse or singular behavior in connection with constant and varying convection, circulating flow patterns or the interplay between convective and reactive terms. In the light of these tests we discuss to what extent the above objectives are met by the current stage of development. Moreover, we identify those issues that require further research. The numerical results confirm that the above principal goal has been achieved, namely that adaptivity combined with scale dependent multigrid ingredients significantly reduces storage and computational complexity and above all also does stabilize Galerkin discretizations even for strongly dominating convection as long as layers are *locally resolved*.

Key Words: Convection dominated problems, stable completions, scale dependent transfer operators and smoothers, full multigrid, adaptive refinements.

AMS Subject Classification: 65M12, 35J20, 76Rxx, 65N30

[*]The work of this author is supported in part by the European Commission within the TMR project (Training and Mobility for Researchers) "Wavelets and Multiscale Methods in Numerical Analysis and Simulation", No. ERB FMRX CT98 018TA

[†]The work of this author is supported in part by the SFB 401 of the Deutsche Forschungsgemeinschaft.

[‡]The work of this author is supported by the Deutsche Forschungsgemeinschaft in the Graduiertenkolleg "Analyse und Konstruktion in der Mathematik" at the RWTH Aachen.