Model Order Reduction Techniques

Problem Set 4 – Geometry Parameters

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Problem Statement – Geometry Parameters

So far we only considered the thermal conductivities and the Biot number as parameters for the thermal fin problem. In PS1 you showed that the bilinear and linear forms satisfy the affine parameter dependence in this case.

We now look at a more realistic (complicated) design problem and assume that you are also free to change the geomtry of the fin itself. More precisely, you are free to adjust the thickness t and length L of the subfins. For simplicity, however, we assume that we vary the thickness and length of all subfins by the same amount. A sketch of the reference geometry of the fin is shown Figure 1. Note that the post is of width unity and height four; the subfins of the reference geometry are of fixed thickness $\bar{t} = 0.25$ and length $\bar{L} = 2.5$.

There are certain constraints you have to obey: the height of the thermal fin and the width of the post is fixed. Also, the top surface of Fin #1 has to remain at height one, of Fin #2 and height 2, of Fin #3 at height 3, and finally of Fin #4 at height 4. As you vary the thickness of the fins, only the lower surface of the fins will thus move.

Our goal is to effectively remove heat from a surface, i.e., the root of the fin. The fin is now characterized by a seven-component parameter vector $\mu = (\mu_1, \mu_2, \ldots, \mu_7) \in \mathcal{D}$, where the first five parameters remain the same as before, i.e., $\mu_i = k^i, \ i = 1, \ldots, 4$, and $\mu_5 = \mathrm{Bi}$; and the sixth and seventh parameter are the thickness and length of the subfins, i.e., $\mu_6 = t$ and $\mu_7 = L$, respectively. The admissible design set is given by $\mathcal{D} \equiv [0.1, 10]^4 \times [0.01, 1] \times [t_{\min}, t_{\max}] \times [L_{\min}, L_{\max}] \subset \mathbb{R}^7$.

Q1. The goal of this problem set is to derive an affine decomposition for the fin given the geometric variations.

α) Due to space requirement, you know that $L_{\min} = 1$ and $L_{\max} = 4$. Give a reasonable range for the thickness t in terms of $t_{\min}$ and $t_{\max}$.

β) Draw a sketch of the fin and clearly mark the partitions required to obtain an affine decomposition of the bilinear and linear forms. (Note that we would like to obtain a conforming finite element triangulation.)

γ) In PS1 you showed that the bilinear and linear forms are given by

$$
\begin{aligned}
a(w, v; \mu) &= \sum_{i=0}^{4} k^i \int_{\Omega^i(t,L)} \nabla w \cdot \nabla v \, dA + \mathrm{Bi} \int_{\Gamma(t,L) \setminus \Gamma_{\mathrm{root}}} wv \, dS, \\
\ell(v) &= \int_{\Gamma_{\mathrm{root}}} v \, dS,
\end{aligned}
$$

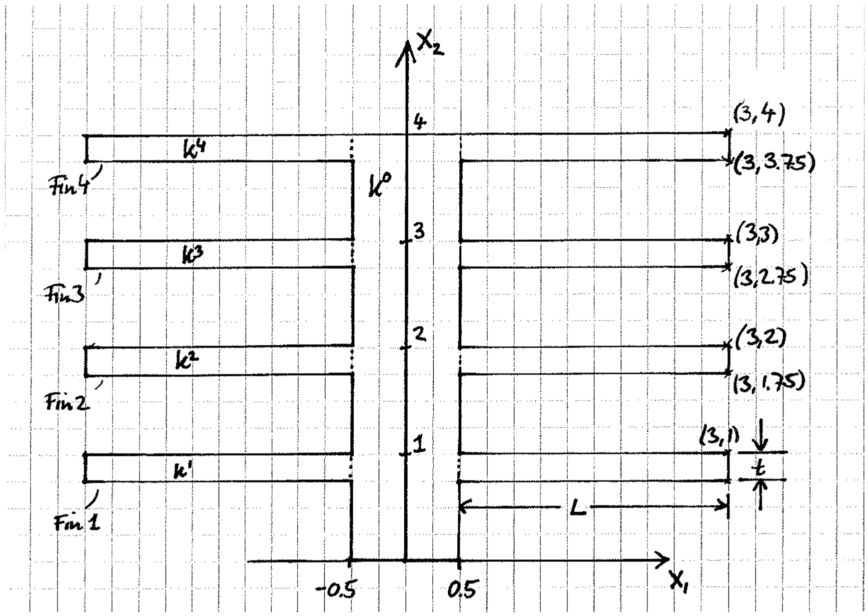

Figure 1: Reference geomtry of thermal fin.

from which the affine decomposition directly followed. However, now the subdomains Ω^i and the surface Γ also depend on the parameters $\mu_6 = t$ and $\mu_7 = L$. Perform a mapping to the reference geometry in order to recover the affine parameter dependence. What is the minimum Q_a required? What are the $\Theta_a^q(\mu)$ and the $a^q(w, v)$?