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NUMERICAL SIMULATION OF LASER-INDUCED CAVITATION BUBBLES

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Outline

1. Investigation in 1d
   (a) Mathematical model
   (b) Two different approaches : Saurel-Abgrall and the level-set function
   (c) Initial data
   (d) Comparison between experiments and approaches
2. Investigation in 2d
   (a) Mathematical model
   (b) Modified real ghost fluid method in 2D
   (c) Correction due to the levelset
   (d) Results of a spherical bubble near a rigid wall
3. Outlook

1d : Mathematical model

- The 1d-Euler equations in spherical coordinates

$$\frac{\partial}{\partial t}(r^2\,\rho) + \frac{\partial}{\partial r}(r^2(\rho\, v_r)) = 0$$
$$\frac{\partial}{\partial t}(r^2\,\rho\, v_r) + \frac{\partial}{\partial r}(r^2(\rho\, v_r^2 + p)) = 2\,p\,r$$
$$\frac{\partial}{\partial t}(r^2\,\rho\, E) + \frac{\partial}{\partial r}(r^2(\rho\, v_r(E + p/\rho))) = 0$$

- The stiffened gas pressure law is used to close the system.

$$p(\rho, e, \varphi) = (\gamma(\varphi) - 1)\rho e - \gamma(\varphi)\pi(\varphi). \tag{1}$$

φ is the phase indicator function (gas fraction, level set function).

Saurel Abgrall Approach

- The two phases (gas and liquid) are distinguished by the mass fraction φ which satisfies a transport equation without mass transfer.

$$\frac{\partial \varphi}{\partial t} + v_r \frac{\partial \varphi}{\partial r} = 0.$$

- For the pure phases, the coefficients γ and π are obtained by measurements.
- A linear interpolation between the two phases is used for the mixture,

$$\begin{aligned} \beta_1(\varphi) &= \varphi \beta_1(1) + (1-\varphi)\beta_1(0), \\ \beta_2(\varphi) &= \varphi \beta_2(1) + (1-\varphi)\beta_2(0). \end{aligned}$$

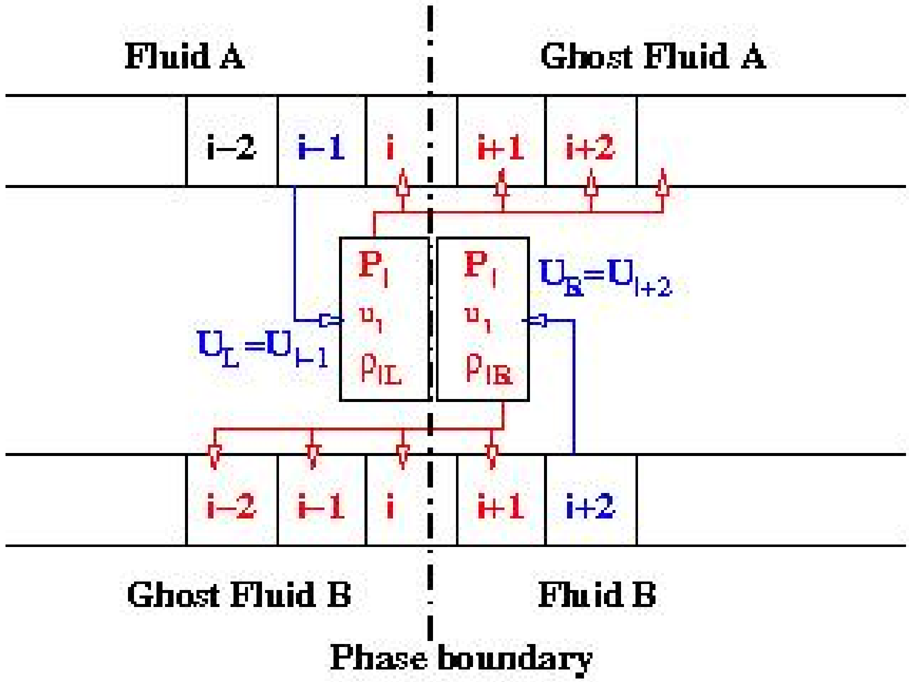
where β_1 and β_2 are defined by $\beta_1 = 1/(\gamma - 1)$ and $\beta_2 = \gamma\pi/(\gamma - 1)$.

Real Ghost Fluid Method (Wang, Liu, Khoo)

- The two phases are distinguished by the function levelset (distance function) which satisfies a transport equation.

$$\partial_t \varphi + \mathbf{v} \,.\, \bigtriangledown \varphi = 0.$$

- A Riemann problem is defined at the interface ($\varphi = 0$) and solved for predicting the interfacial states (ρ_{IL}, ρ_{IR}, p_I and u_I).

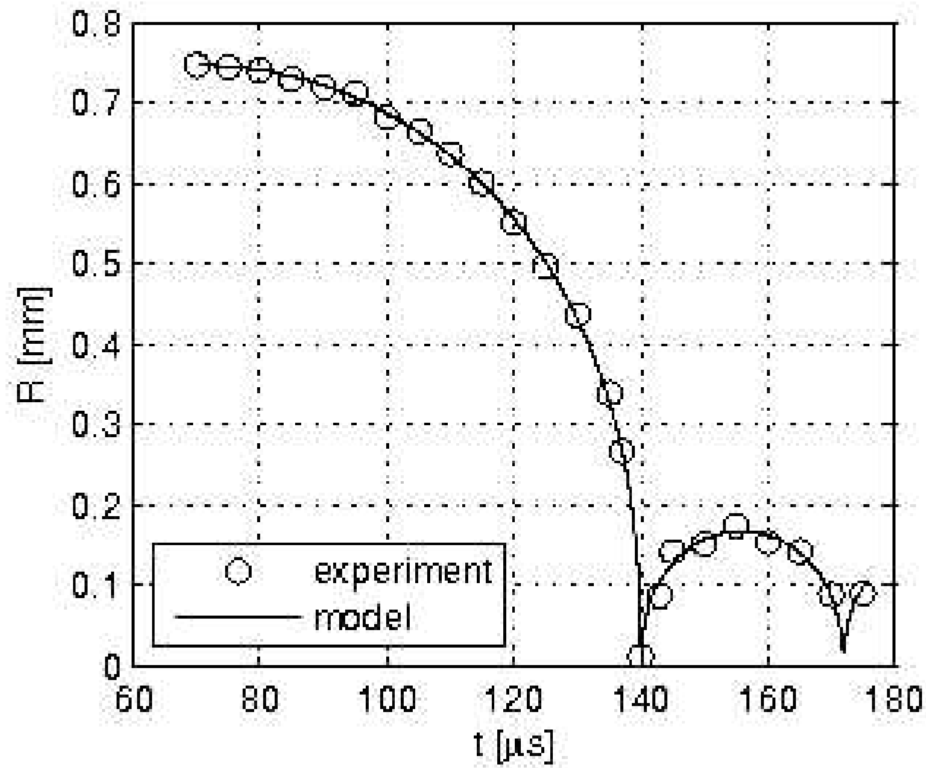


Initial Data : Fitting of Equilibrium Radius

Initial conditions :

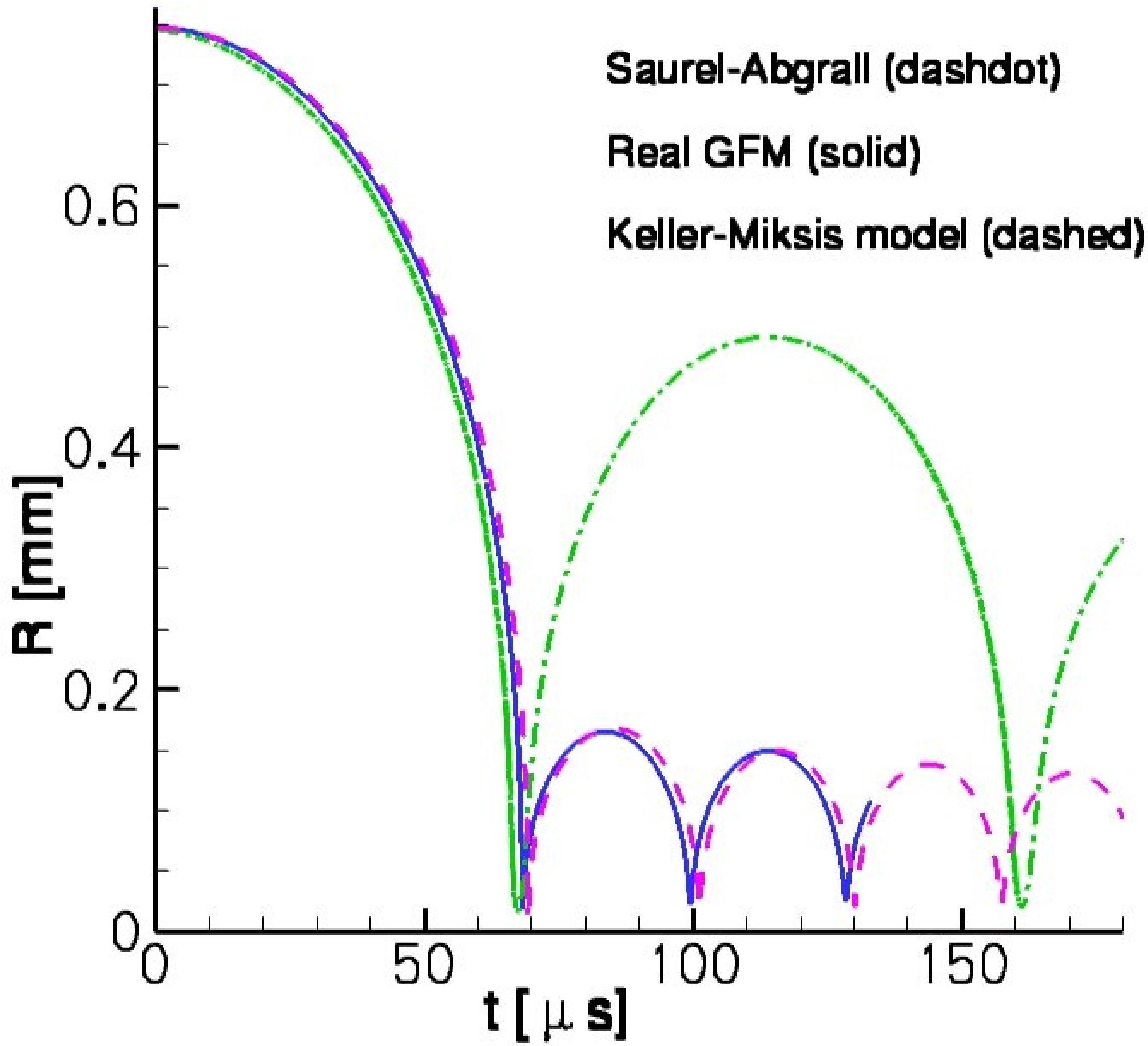
- $t_{max} = 70.7\ \mu s$ ("Exp")
- $R_b = R_{max}$ (Exp)
- $\dot{R}_b = 0$

$\Rightarrow R_{eq} = 6.92 \times 10^{-5}$ m in minimizing the least square error of the Keller-Miksis model.



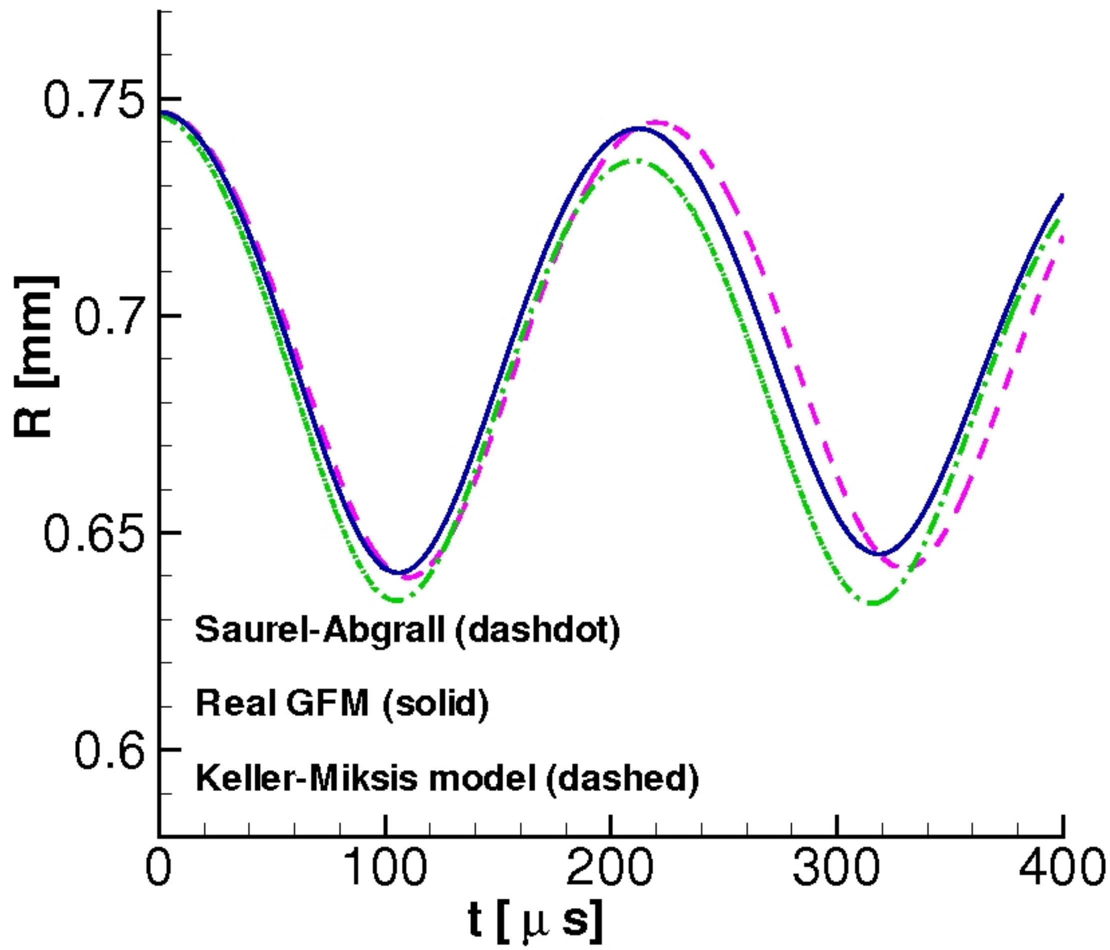
	Initial data		Material parameters			
	ρ [kg/m^3]	p [Pa]	γ [-]	π [Pa]	c_v [J/kg K]	$\mathcal{R}$ [J/kg K]
Gas	9.5e-4	4.57	1.4	0	708.3	283.32
Liquid	1000	100000	1.1	2.e+9	4190.0	418

Numerical Results: R$_{eq}$ = 6.92 x 10^{-5} m



Numerical Results: $R_{eq} = 6.92 \times 10^{-4}$ m

	Initial data		Material parameters			
	ρ [kg/m^3]	p [Pa]	γ [-]	π [Pa]	c_v [J/kg K]	$\mathcal{R}$ [J/kg K]
Gas	9.57e-1	72560	1.4	0	708.3	283.32
Liquid	1000	100000	1.1	2.e+9	4190.0	418



2d : Mathematical model

- The 2d-Euler equations with rotational symmetry

$$
\begin{aligned}
&\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho\, v_r) + \frac{\partial}{\partial z}(\rho\, v_z) + \frac{1}{r}\rho\, v_r = 0 \\
&\frac{\partial}{\partial t}(\rho\, v_r) + \frac{\partial}{\partial r}(\rho\, v_r^2 + p) + \frac{\partial}{\partial z}(\rho\, v_r\, v_z) + \frac{1}{r}\rho\, v_r^2 = 0 \\
&\frac{\partial}{\partial t}(\rho\, v_z) + \frac{\partial}{\partial r}(\rho\, v_r\, v_z) + \frac{\partial}{\partial z}(\rho\, v_z^2 + p) + \frac{1}{r}\rho\, v_z\, v_r = 0 \\
&\frac{\partial}{\partial t}(\rho\, E) + \frac{\partial}{\partial r}(v_r(\rho\, E + p)) + \frac{\partial}{\partial z}(v_z(\rho\, E + p)) + \frac{1}{r}v_r(\rho\, E + p) = 0
\end{aligned}
$$

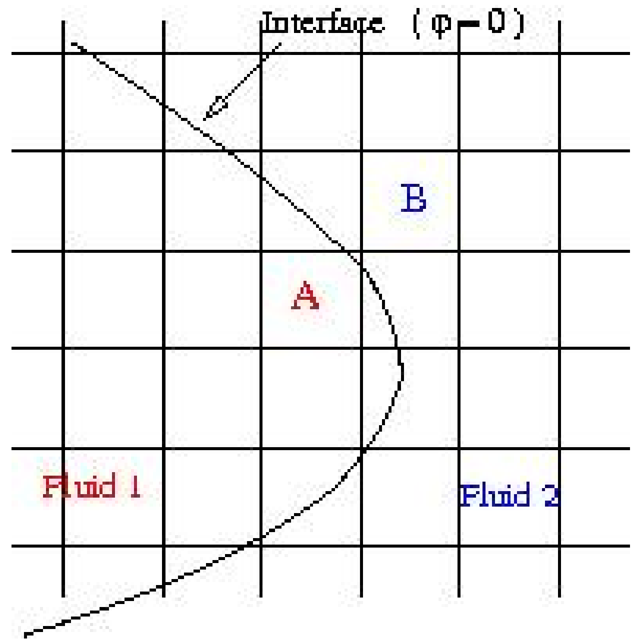
- The stiffened gas pressure law is used to close the system.
  $p(\rho, e, \varphi) = (\gamma(\varphi) - 1)\rho e - \gamma(\varphi)\pi(\varphi)$ with φ the level-set function.

Modified Real Ghost Fluid Method

- The levelset defines the normal to the interface $N = \frac{\nabla\varphi}{|\nabla\varphi|}$

  - A Riemann problem is defined at the interface between the cell A and B such that the angle made by the respective normals are the minimum

  - The interfacial states define the ghost cells (ρ_{IL}, ρ_{IR}, p_I and u_I).



- A single-phase Riemann problem is solved at cell interfaces with ghost cells as boundary condition.

- The solution can be advanced to the next time step.

Change of the levelset sign in a cell after update

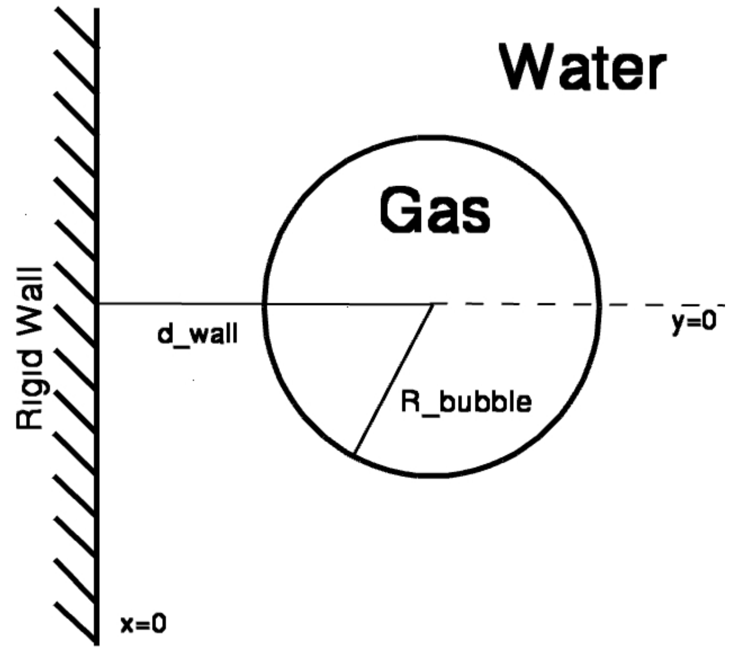
- Modification of the energy using the corresponding gas law.

$$\mathbf{U_k}^{n+1} = (\varrho, \varrho v, \varrho E) \rightarrow p\,(\varrho, e, EOS(\phi_k^n)) \rightarrow p\left(\varrho, p, EOS(\phi_k^{n+1})\right) \\ \rightarrow \tilde{\mathbf{U}}_{\mathbf{k}}^{n+1} = \left(\varrho, \varrho v, \varrho\tilde{E}\right).$$

- Modification of the density with the corresponding ghost cell.

$$\tilde{\mathbf{U}}_{\mathbf{k}}^{n+1} = \left(\varrho, \varrho v, \varrho\tilde{E}\right) \rightarrow \tilde{\mathbf{U}}_{\mathbf{k}}^{n+1} = \left(\varrho^{GC}, \varrho^{GC} v, \varrho^{GC}\tilde{E}^{GC}\right).$$

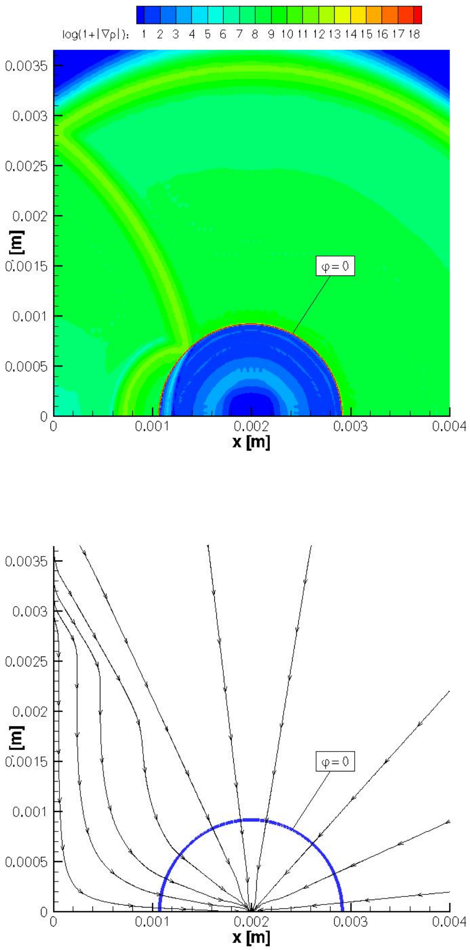
Results : Spherical bubble near a rigid wall



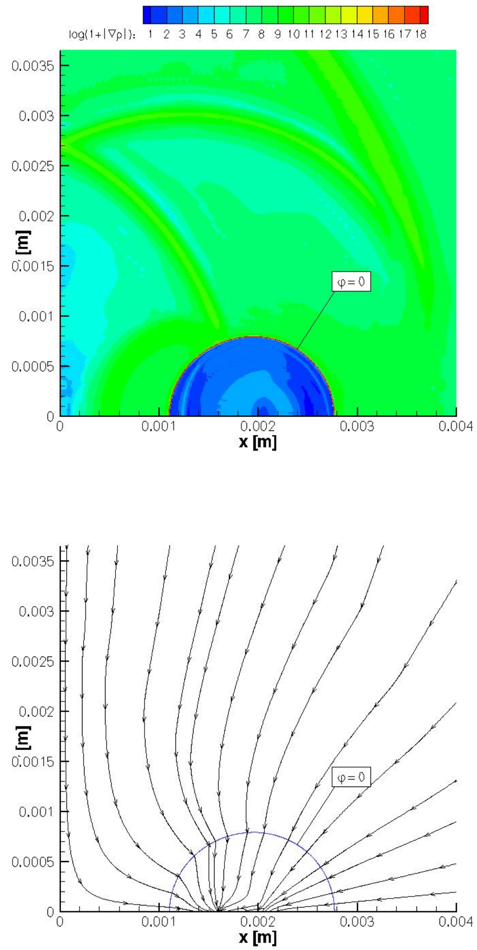
- Initial data

	Initial data		Material parameters			
	ρ [kg/m^3]	p [Pa]	γ [-]	π [Pa]	c_v [J/kg K]	$\mathcal{R}$ [J/kg K]
Gas	0.026077	2118	1.4	0	717.5	283.32
Liquid	1000	50000000	7.15	3.e+8	201.1	1236.765

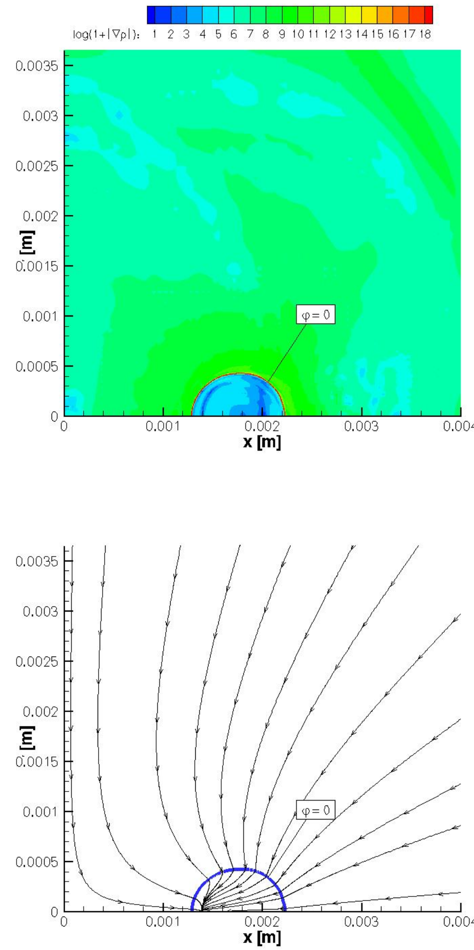
Results : Spherical bubble near a rigid wall



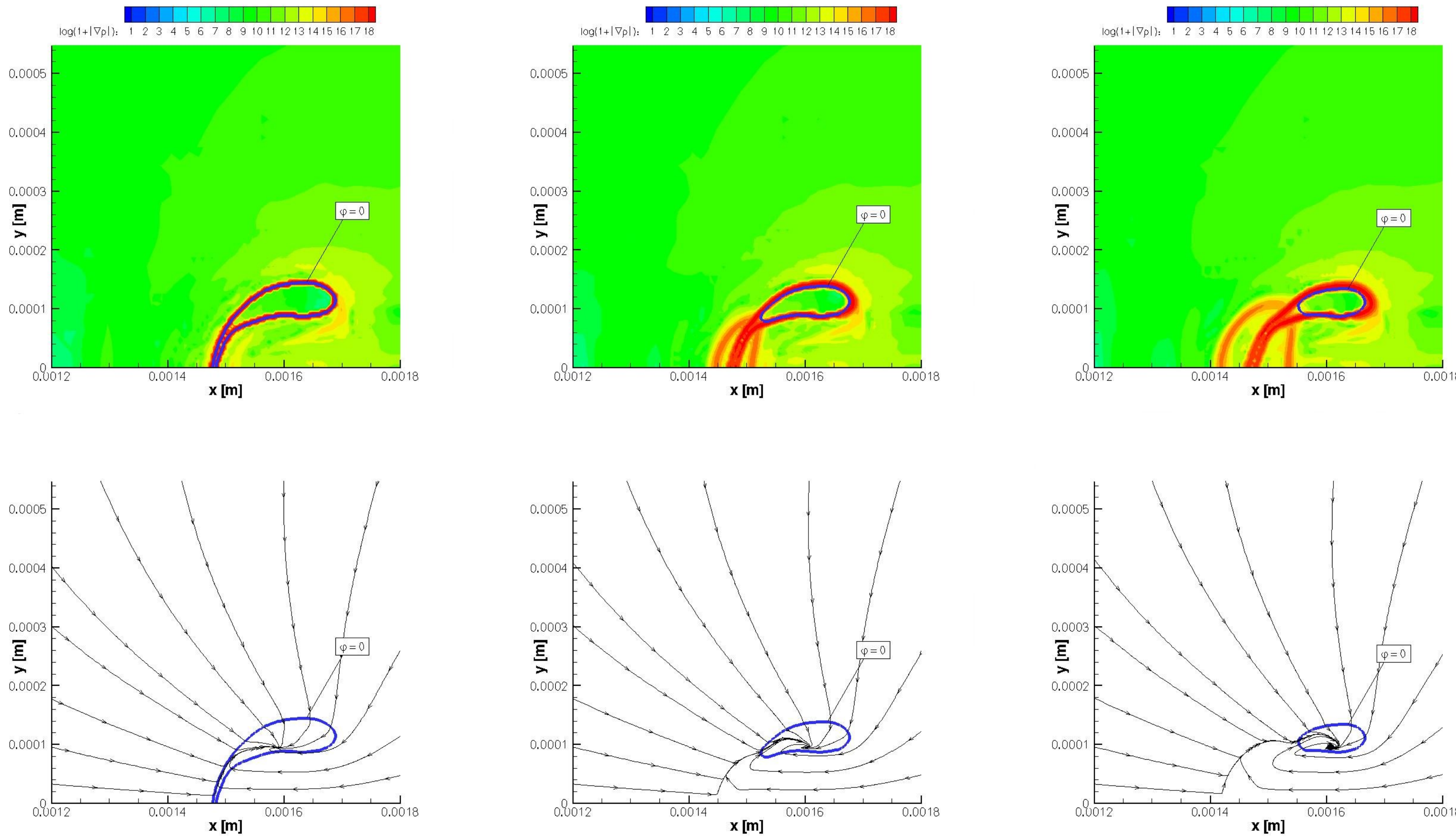
t = 1.59 μs



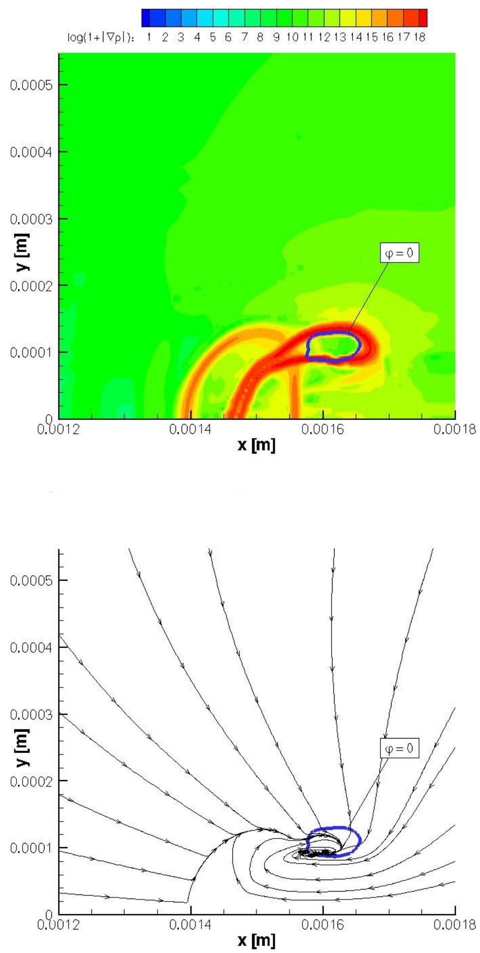
t = 3.25 μs



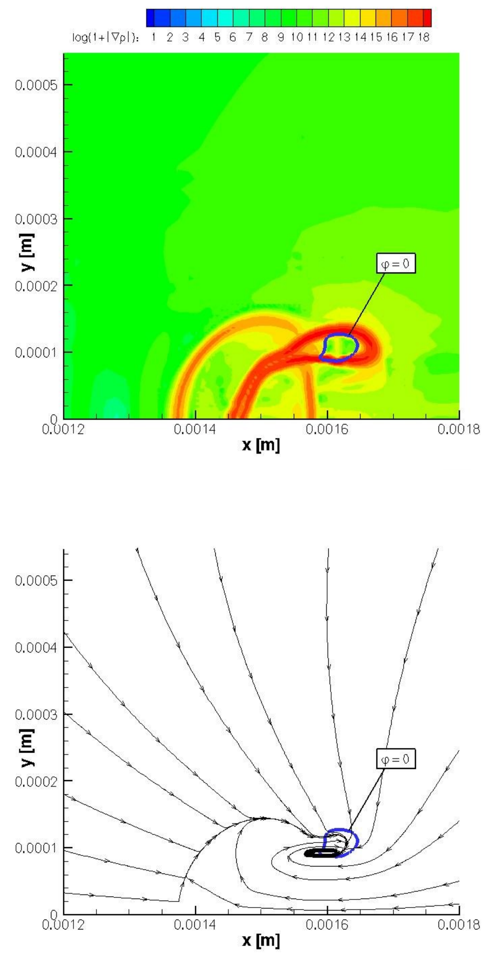
6.50 μs



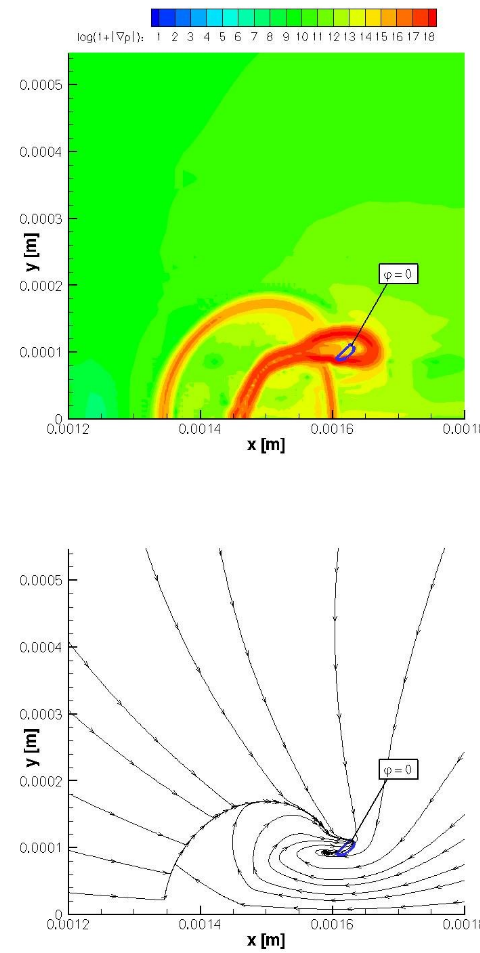
t = 8.01 μs   t = 8.14 μs   8.16 μs



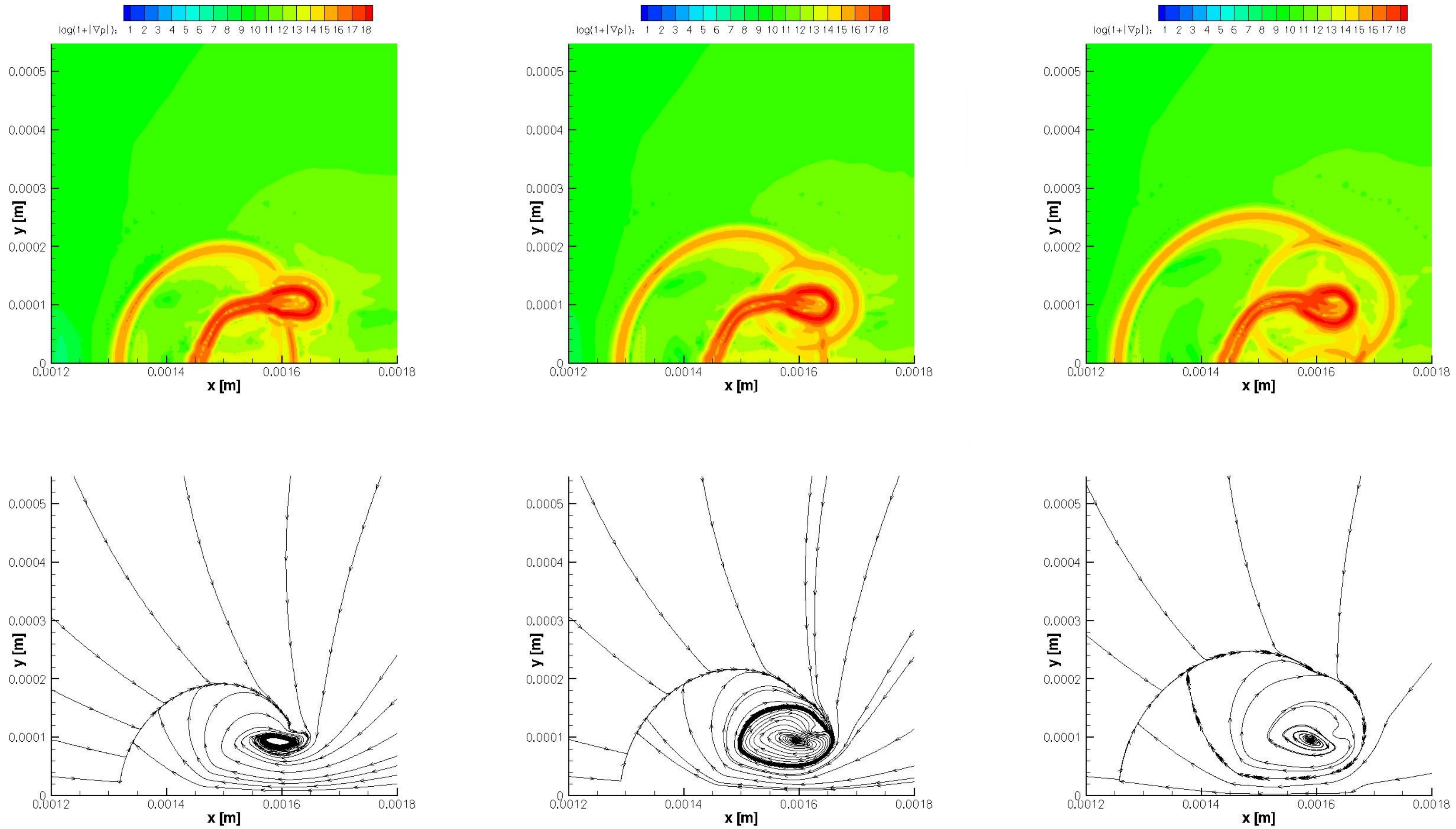
t = 8.17 μs



t = 8.19 μs



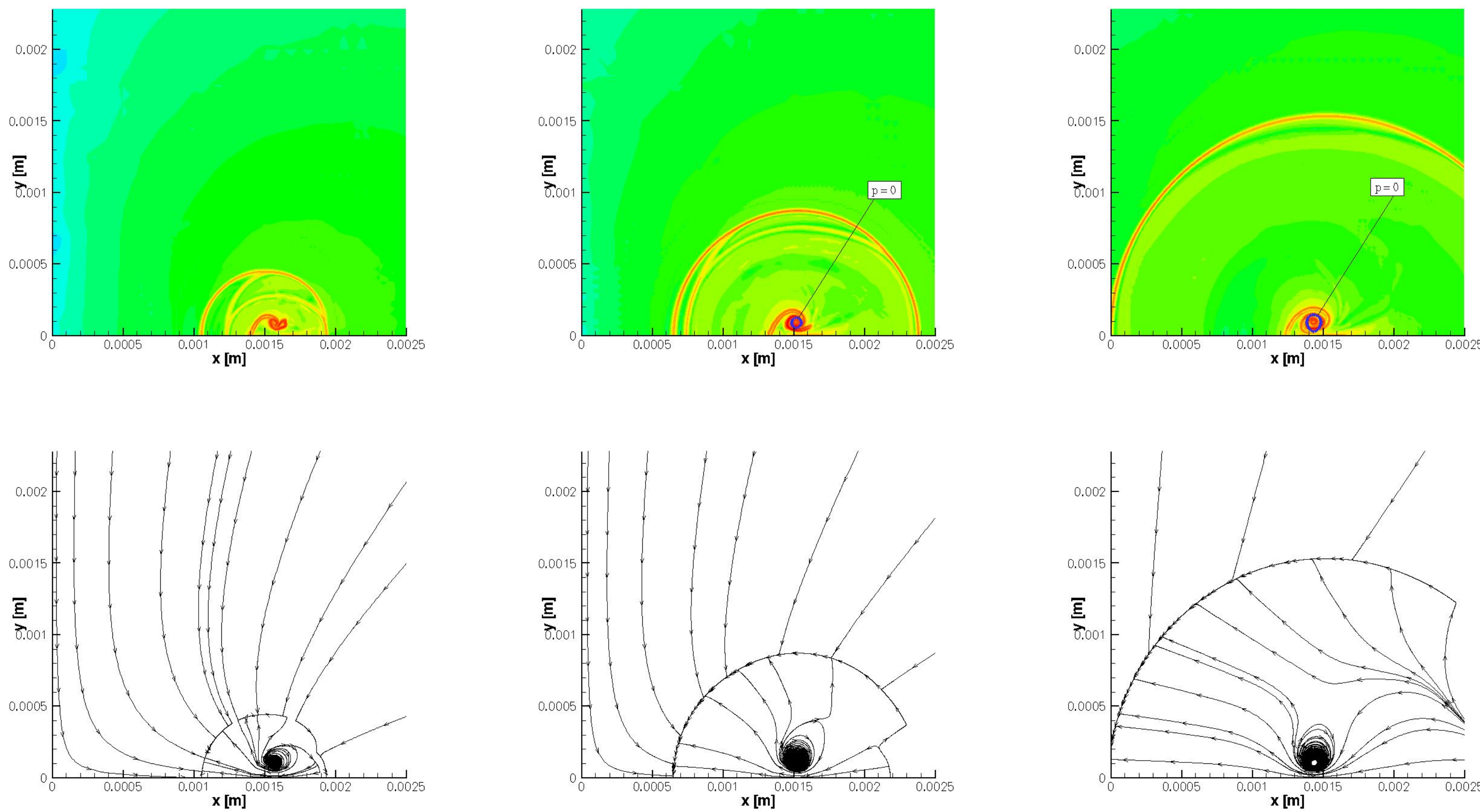
t = 8.20 μs



t = 8.21 μs

t = 8.23 μs

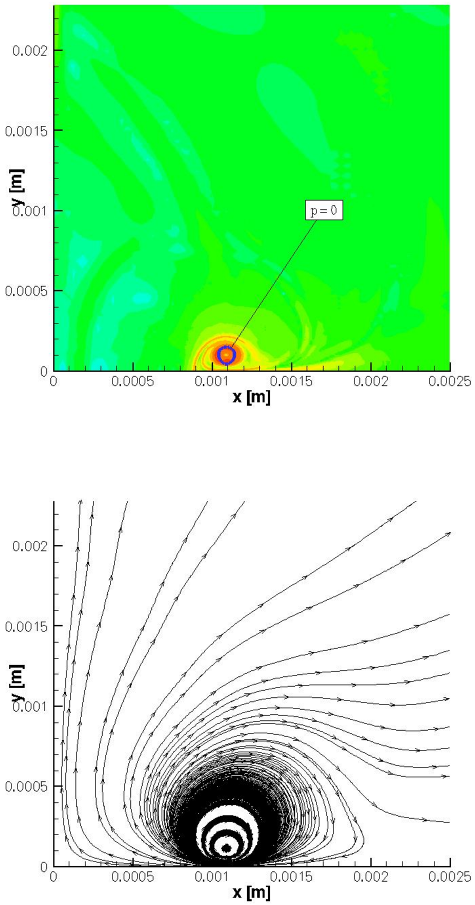
t = 8.24 μs



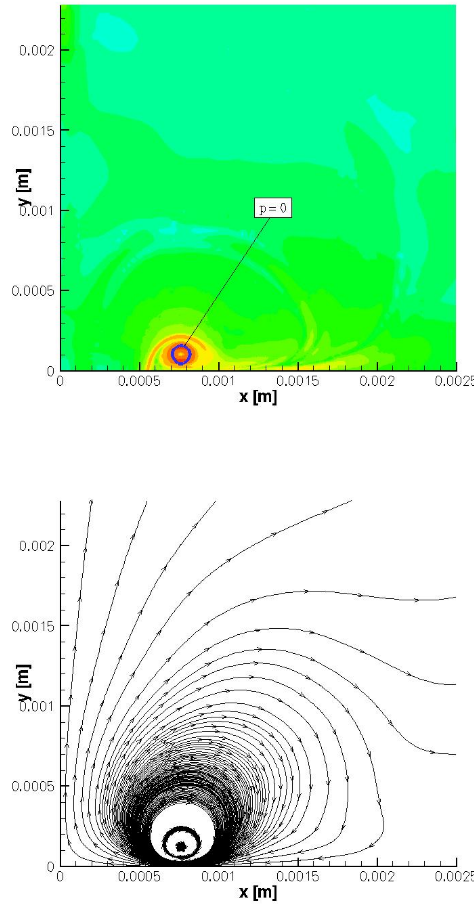
t $= 8.34\ \mu$s

t $= 8.55\ \mu$s

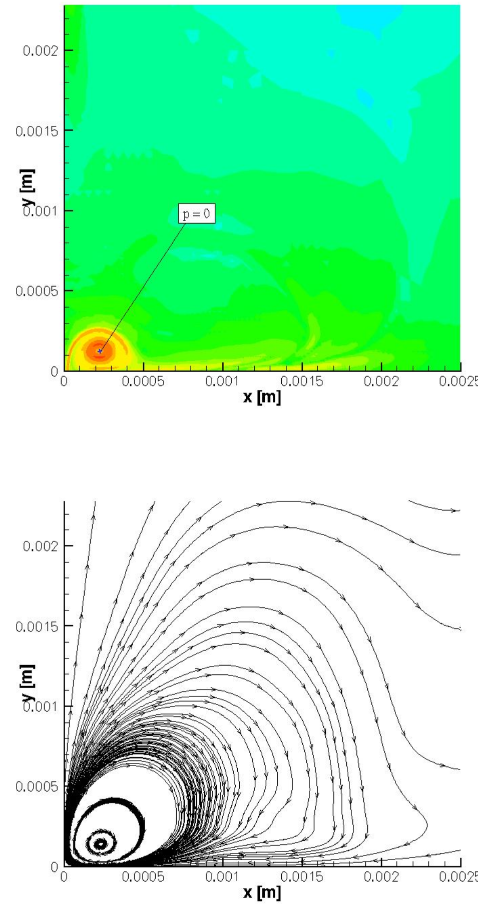
t $= 8.92\ \mu$s



t = 11.07 μs



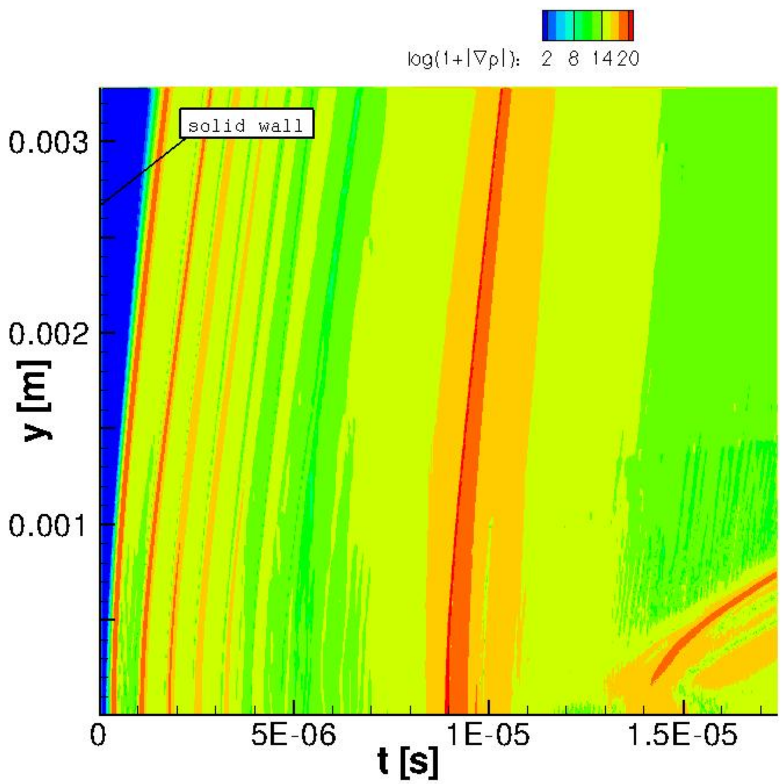
t = 13.36 μs



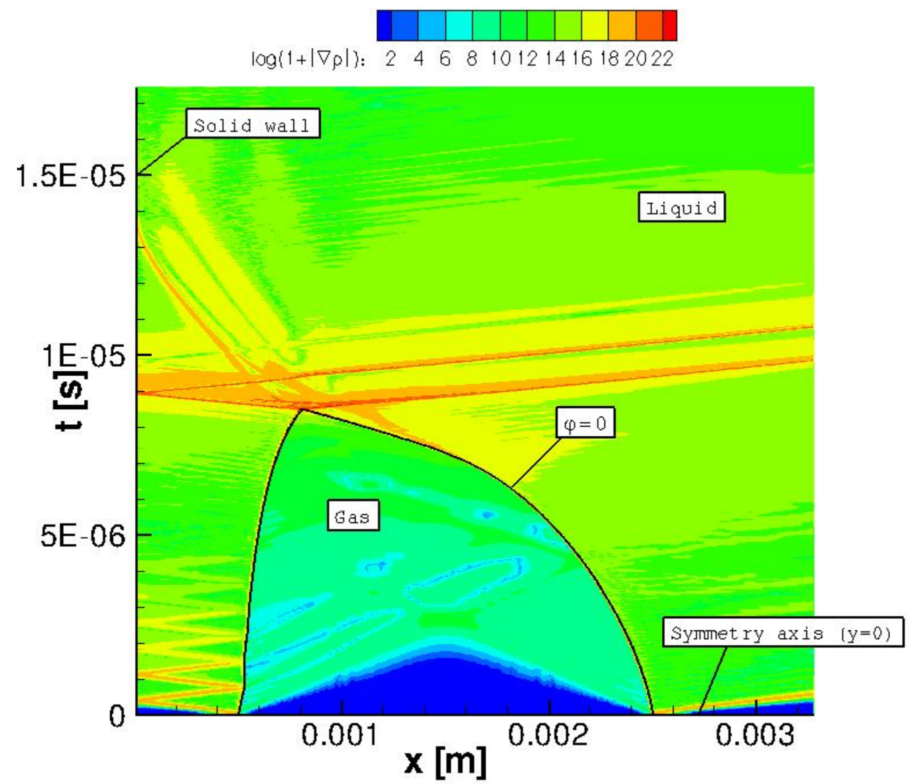
t = 18.03 μs

Results : Spherical bubble near a rigid wall

Extract along the wall



Extract along the symmetry axis



Conclusion

Saurel-Abgrall:

- Severe numerical phase transition
- Rebound overpredicted

Real Ghost Fluid Method:

- No phase transition
- Rebound well-predicted

Real Ghost Fluid Method in 2D:

- Difficulties to maintain the interface sharp due to the geometry and the extreme conditions
- Loss of the gas phase

Future Work

- Investigation of shock bubble interactions.

- Qualitative comparisons with experimental data.