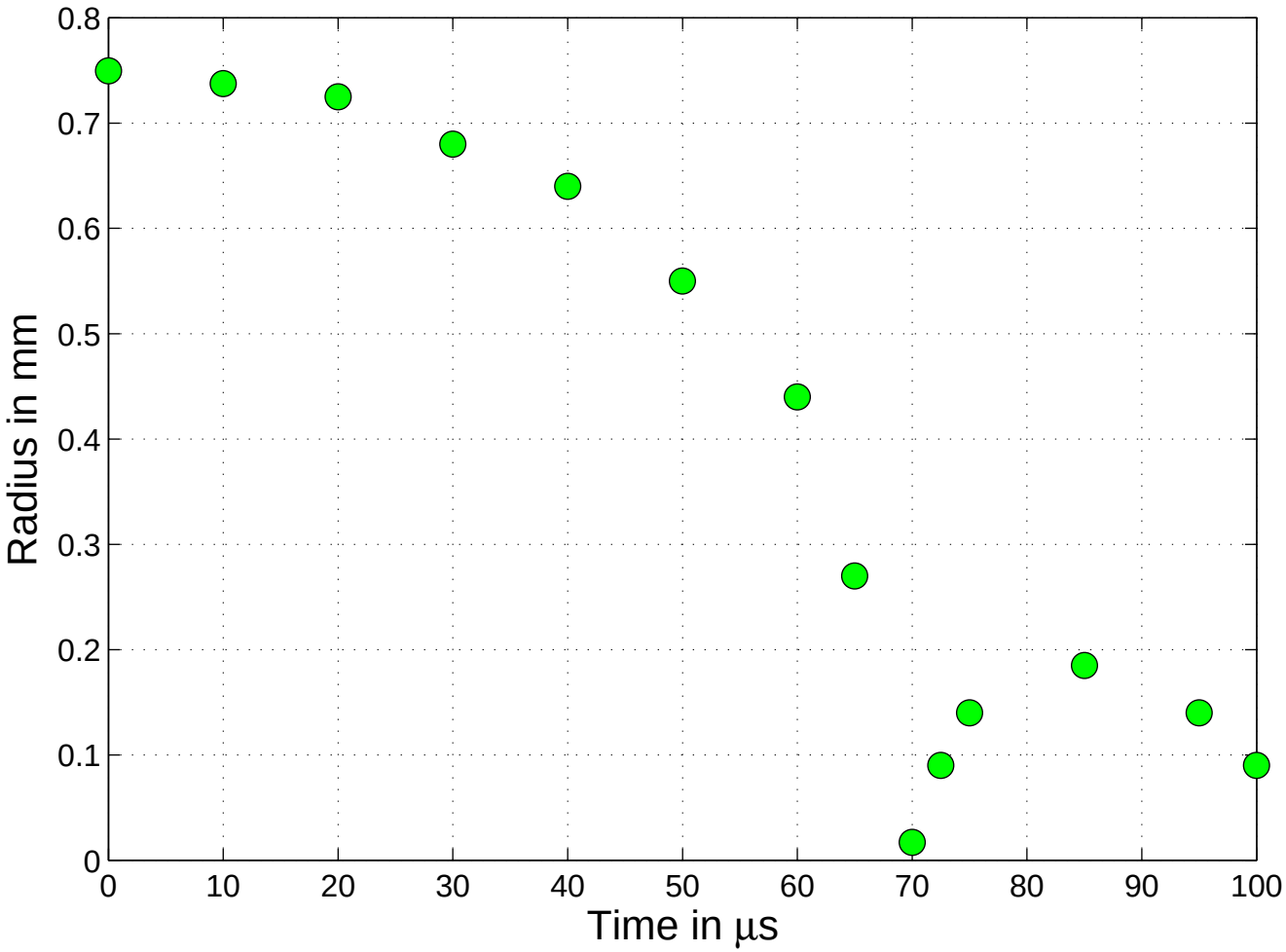
Bubble dynamics in a compressible liquid with phase transition

Wolfgang Dreyer Frank Duderstadt
Maren Hantke Gerald Warnecke

WIAS Berlin
Otto-von-Guericke-University Magdeburg

Micro-Macro Modelling and Simulation of Liquid-Vapour Flows
Aachen, February 2009

A laser-induced cavitation bubble – experimental data



- The bubble consists of water vapour, hydrogen and oxygen.
- Homogeneity in the gas phase
- Ideal gas

- Incompressible liquid
- Isotherm process

$$F = R^2 \dot{R}$$

$$\dot{F} = \frac{F^2}{2R^3} - \frac{R}{\rho^L}(p_0 - p^L)$$

Second case

- The bubble consists of water vapour, hydrogen and oxygen.
- Homogeneity in the gas phase
- Ideal gas
- Incompressible liquid
- Isotherm process
- Phase transition

$$
\begin{aligned}
\dot{m} &= 4\pi R^2 m_0 \frac{m_0}{kT} \frac{p^W}{\sqrt{2\pi m_0 kT}} (g^L - g^V + [\![e_{kin}]\!]) \\
F &= R^2 \dot{R} - \frac{\dot{m}}{4\pi\rho^L} \\
\dot{F} &= \frac{F^2}{2R^3} - \frac{R}{\rho^L}(p_0 - p^L)
\end{aligned}
$$

Third case

- The bubble consists of water vapour, hydrogen and oxygen.
- Homogeneity in the gas phase
- Ideal gas
- Incompressible liquid
- Phase transition
- Heat conduction

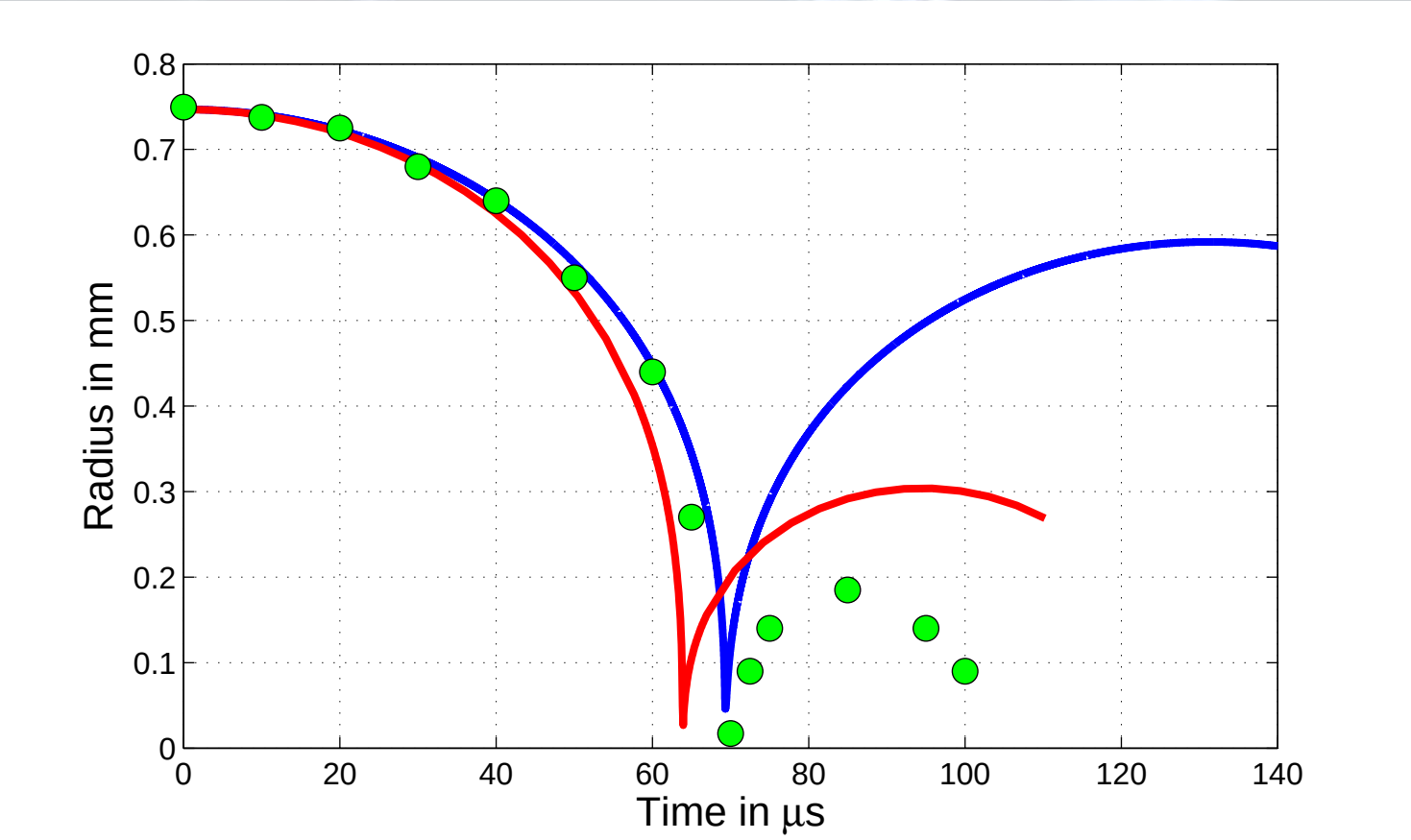
$$
\begin{aligned}
\dot{m} &= 4\pi R^2 m_0 \frac{m_0}{kT} \frac{p^W}{\sqrt{2\pi m_0 kT}} (g^L - g^V + [\![e_{kin}]\!]) \\
F &= R^2 \dot{R} - \frac{\dot{m}}{4\pi\rho^L} \\
\dot{F} &= \frac{F^2}{2R^3} - \frac{R}{\rho^L}(p_0 - p^L) \\
\frac{\partial T^L}{\partial t} + v^L \frac{\partial T^L}{\partial r} &= a^L \left(\frac{\partial^2 T^L}{\partial r^2} + \frac{2}{r} \frac{\partial T^L}{\partial r} \right) \\
\frac{\partial T}{\partial r} &= \frac{\lambda \dot{m}}{4\pi R^2 \kappa^L} + \frac{p^V R}{3\kappa^L} \frac{\partial}{\partial t} \log \left(\frac{4\pi (T^V R)^3}{3m} \right) .
\end{aligned}
$$

Proposed model

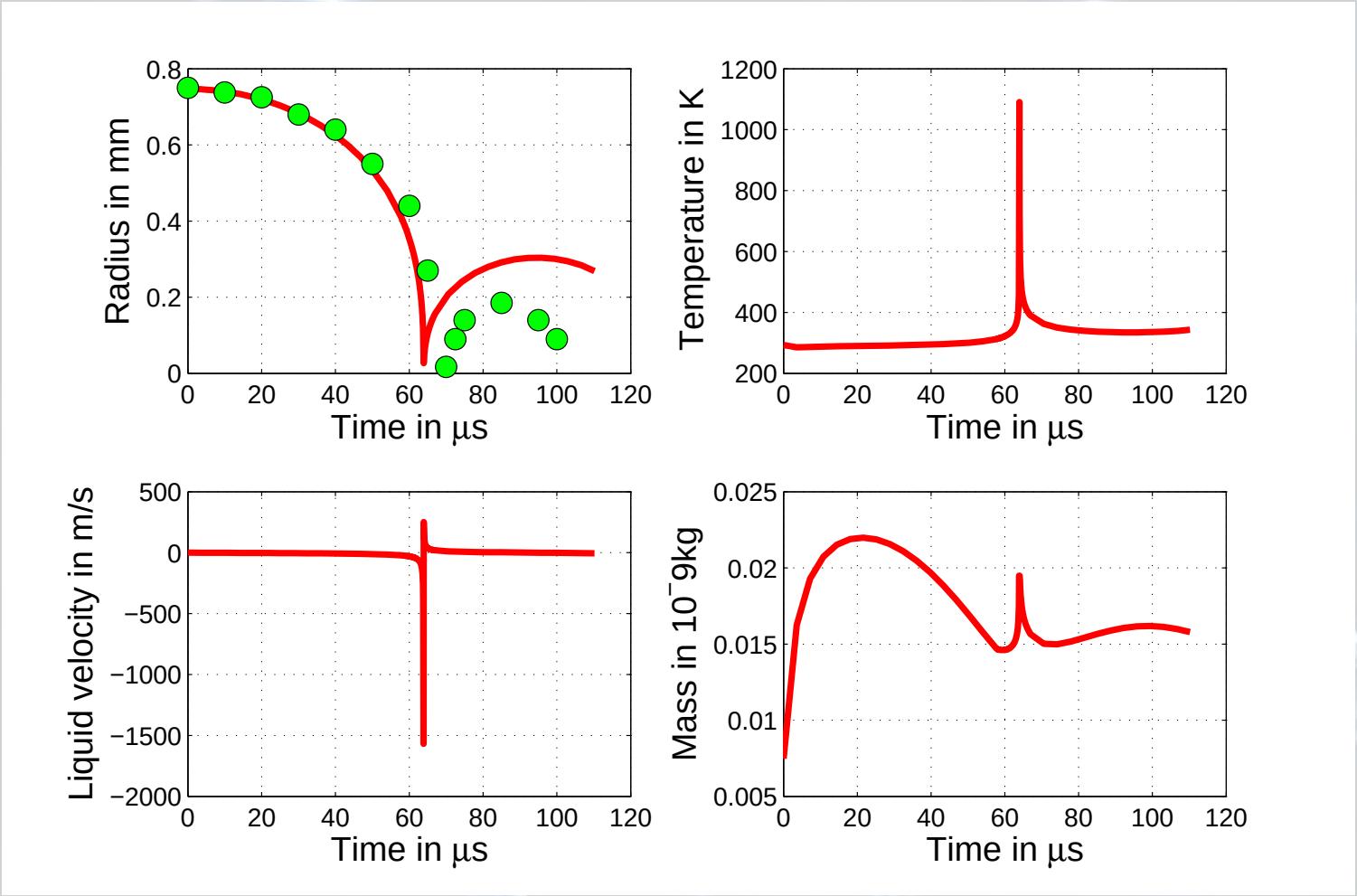
- The bubble consists of water vapour, hydrogen and oxygen.
- Homogeneity in the gas phase
- Ideal gas
- Compressible liquid
- Phase transition
- Heat conduction

$$
\begin{aligned}
\dot{m} &= 4\pi R^2 m_W \frac{m_W}{kT} \frac{p^W}{\sqrt{2\pi m_W kT}} (\mu^L - g^V + [\![e_{kin}]\!]) \\
\phi(R - c_0 t) &= \phi'(R - c_0 t) \cdot R - \dot{R} R^2 + \frac{\dot{m}}{4\pi\rho_0} \\
\phi'(R - c_0 t) &= \frac{R}{\rho_0 c_0} (p_I^L - p_0) \\
\frac{\partial T^L}{\partial t} + v^L \frac{\partial T^L}{\partial r} &= a^L \left(\frac{\partial^2 T^L}{\partial r^2} + \frac{2}{r} \frac{\partial T^L}{\partial r} \right) \\
\frac{\partial T^L}{\partial r} &= \frac{\lambda \dot{m}}{4\pi R^2 \kappa^L} + \frac{p^V R}{3\kappa^L} \frac{\partial}{\partial t} \log \left(\frac{4\pi (T^V R)^z}{3m} \right) .
\end{aligned}
$$

Numerical results

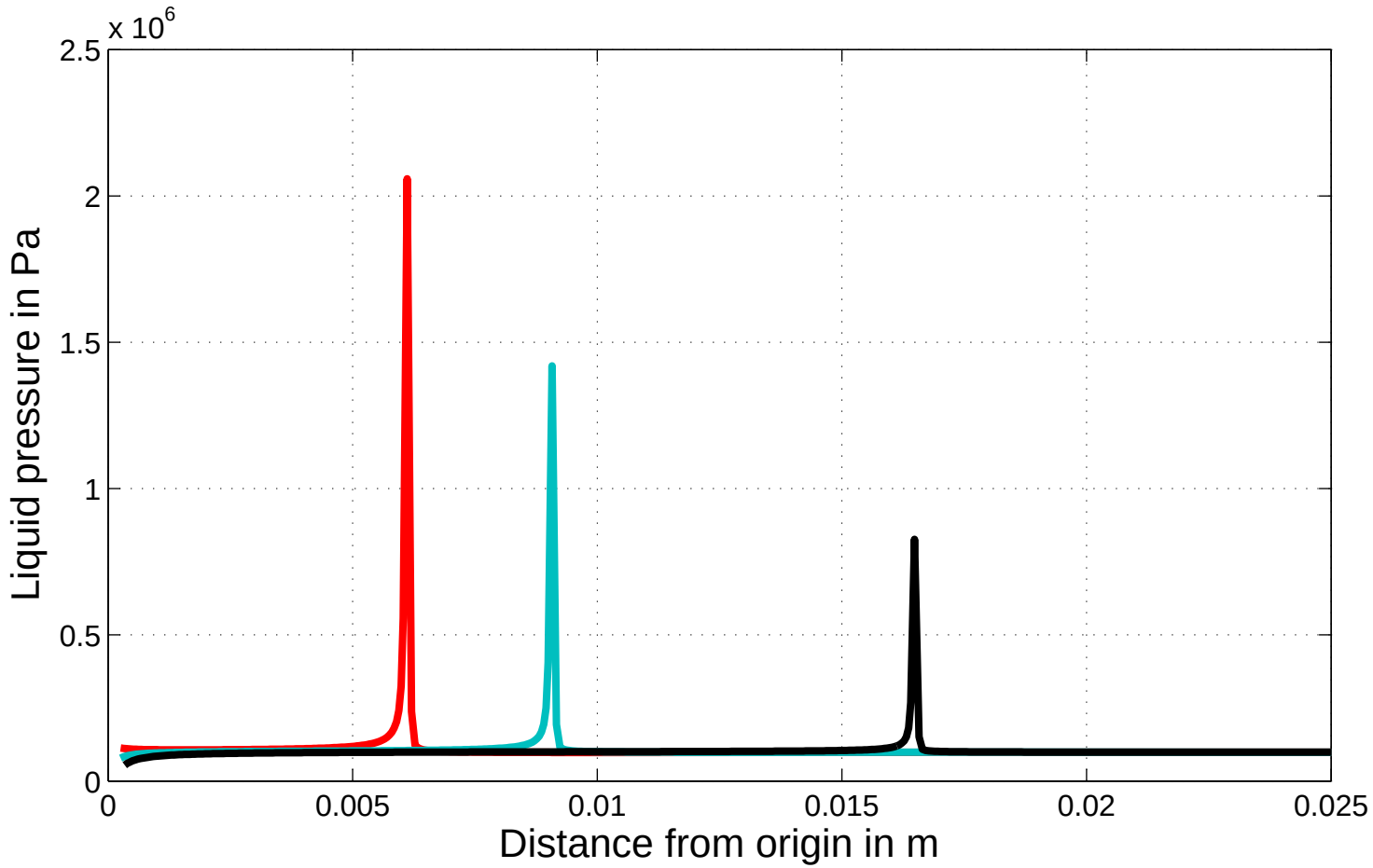


Numerical results



Pressure wave

4 / 6 / 11.5 * 10^-6 s after collapse



Effects we didn‘t take into account / Future work

- thermal extension
- oxygen dissolves in water
- higher order terms for compressibility
- wave propagation in the bubble
- reflecting boundary / velocity field in the beginning

- Multi bubble system
- System of balance laws for mixture theories
- Model non-conservative/production terms
- Pressure less gas model