

Exercise sheet 10 for Friday, Jan 13, 2016

To be handed in either at the beginning of the exercise session, or before Jan 13, 9:55 a.m. at the drop box in front of room 149.

Exercise 35. Let $V_j \subset \mathcal{H}$ be a nested sequence of closed subspaces with associated projectors $Q_j : \mathcal{H} \rightarrow V_j$. Show that the following properties are equivalent:

(i) The projectors commute

$$Q_j Q_k = Q_k Q_j, \quad j, k \in \mathbb{N}_0.$$

(ii) The operators $Q_{j+1} - Q_j$ are also projectors.

(iii) The ranges $\tilde{V}_j := \text{range } Q_j^*$ are also nested

$$\tilde{V}_j \subset \tilde{V}_{j+1}, \quad j \in \mathbb{N}_0,$$

where Q_j^* are the adjoints of Q_j .

Show that as a consequence of these properties one has

$$(Q_{j+1} - Q_j)(Q_{k+1} - Q_k)f = ((Q_{j+1} - Q_j)f)\delta_{jk} \quad \forall j, k \in \mathbb{N}_0.$$

8+2 points