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Finite Volumen und Finite Elemente Verfahren II - WS 18

1. Exercise sheet

Attention: This sheet is a warm-up exercise, in which you get familiar with the three prototypes underlying much of the development of the theory of hyperbolic conservation laws. This sheet will not be discussed in the exercise and does not count for the access to the oral exam. You can find many details of these equations and hyperbolic conservation laws in, e.g., [1, 2, 3, 4].

Ex 1: Eigenstructure

Write down the conservative variables and fluxes $(U, F(U))$ for the following three systems. Compute the Jacobian matrix $F'(U) = A(U) \in \mathbb{R}^{m\times m}$, its eigenvalues $\lambda_k(U)$ and eigenvectors $r_k(U)$ for $k = 1 \dots m$ either by hand or using a computer-algebra system.

a) Burgers' equation

$$u_t + \left(\frac{1}{2}u^2\right)_x = 0.$$

b) The isentropic Euler equations

$$\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2 + k\rho^\gamma)_x &= 0 \end{aligned}$$

with real constants $k > 0$, $\gamma > 1$.

c) The full Euler equations

$$\begin{aligned} \rho_t + (\rho u)_x &= 0 \\ (\rho u)_t + (\rho u^2 + p)_x &= 0 \\ E_t + ((E + p)u)_x &= 0 \end{aligned}$$

with polytropic equation of state $E = \frac{p}{\gamma-1} + \frac{1}{2}\rho u^2$, and $\gamma > 1$. Let $c > 0$ be given by $c^2 = \frac{\gamma p}{\rho}$. Argue that c is the speed of sound.

Literatur

[1] R. Courant and K. Friedrichs and H. Lewy, *Über die partiellen Differenzengleichungen der mathematischen Physik*, Mathematische Annalen, 1928.

[2] C. M. Dafermos, *Hyperbolic Conservation Laws in Continuum Physics*, Springer Berlin / Heidelberg, 2005.

[3] R. J. LeVeque, *Numerical Methods for Conservation Laws*, Birkhäuser Basel, 1990.

[4] J. Smoller, *Shock Waves and Reaction-Diffusion Equations*, Springer New York, 1994.