Figure 6: no f ∥ 37 ℓ which can be seen as the reconstructed.

Computer tomography, e.g., in hospitals, gives pictures from the interior of objects using X-ray beams. The same principle holds for electron tomography for objects on a microscopical scale; where the specimen is scanned by electron beams. In an electron microscope, measurements of the transmitted electrons provide an intensity distribution. Using this data the two dimensional pictures of cross sections of the object will be reconstructed.

In electron tomography the objects of interest are often of organic nature. To ensure the transmission of electrons, the object has to be very thin, say less than 1 mm (1 mm = 10^-3 m). For technical reasons only measurements from a small range of angles are possible. In addition it is important to reduce the intensity of the electron beam, otherwise the sensitive specimen can be destroyed during the exposure. This leads to the question: What is the lowest electron dose we can choose to still get good results?

From now on we will take a look at an abstract setting. There are two approaches to reconstruct the cross sections. The standard one uses the Fourier slice theorem. However, we will use the second method, which is called algebraic reconstruction technique and leads to a system of linear equations.

\[ \mathbf{A} \mathbf{x} = \mathbf{b} \] (1)

The matrix \( \mathbf{A} \in \mathbb{R}^{m \times N} \) depends on the angles and positions of the electron beams. The measured data itself can be found in \( \mathbf{b} \in \mathbb{R}^m \). The wanted pixel values \( x_{ij} \) of the picture of one slice plane of row \( i \) and column \( j \) are stored in the unknown vector \( \mathbf{x} \in \mathbb{R}^N \). The dimension \( N \gg n \) of this problem can be huge.

The method of choice usually is Kaczmarz's algorithm, which minimizes the residual \( \mathbf{A} \mathbf{x} - \mathbf{b} \) in the \( \ell_2 \)-norm. If the system of linear equations is under-determined, the iterations converge to the solution with the smallest \( \ell_2 \)-norm. In many cases the results are unsatisfying, see Figures 1–4.

Sequential Subspace Optimization

Given a convex and smooth function \( f: \mathbb{R}^N \to \mathbb{R} \), which in our case is a smooth version of the function in (2), the goal is to solve the problem

\[ \arg \min_x f(x). \] (3)

The idea of the sequential subspace optimization (SESOP) [3, 4] arises from the conjugate gradient (CG) method, which is an iterative method for minimizing quadratic functions. Beginning from the last iteration, the function will be minimized in a given direction, see Figure 2 on the left. If the function is quadratic, the CG method provides the exact solution after finitely many iterations.

In some cases, SESOP might be a very good alternative in electron tomography, where only few measurements are available. There are many options to improve the results, like modifying the subspace and optimizing the method for solving the subproblems.

References


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