Grinding Wheel Modeling:
Development of a mathematical Model

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Abstract
Grinding wheels are complex tools with a structure composed of geometrically undefined abrasive grain material, a bonding material and pores. The statistically shaped grains and their distribution define the performance of the tool. Detailed models of the grinding wheel structure do not exist. A model based design of grinding wheels has a great potential to reduce cost and time consuming experiments to identify a suitable grinding wheel composition. This paper describes an approach to model the structure of grinding wheels, based on an analysis of real abrasive grains. A new methodology to evaluate the grain shapes based on computer tomography scan data using a marching cubes algorithm is presented. Besides, a basic model to visualize the structure of volumetric structure elements of a grinding wheel is shown using randomized polyhedrons, calculating the distribution of the grains and the collision distance. The presented work is a first step to generate a realistic model of complex grinding wheel structures based on the composition of the abrasive layer.

Keywords: Grinding Wheel Modeling, Granular Structures

1. State of the art
The manufacturing process grinding is used to a great variety of different workpiece materials and applications. Grinding is not only applied as a finishing process to achieve high workpiece surface qualities. Also, high material removal rates and highly productive processes are possible. Depending on the workpiece
material, the machine tool capabilities and the requested workpiece properties after grinding define the necessary properties of the used grinding tool. Mostly grinding wheels are used which are complex high precision tools designed for specific machining operations. Grinding tools contain the three major components: Abrasive grain material, bonding material and pores.

A grinding wheel and its structure is shown in Figure 1. The chip removal is accomplished by an amount of geometrical undefined abrasive grains plowing through the workpiece material with a relative speed resulting from the circumferential speed $v_s$ of the grinding wheel and the workpiece velocity $v_w$.

As abrasive grain material mainly four different materials are used. Conventional grain materials are corundum and silicon carbide, as the so called super abrasive materials are cubic boron nitride (CBN) and diamond. The important properties of the abrasive grain materials are a high degree of hardness and toughness as well as high thermal and chemical resistance [6].

The shape of the grains is geometrical undefined while the average size can vary from a sub micrometer range up to 500 $\mu$m. Before the grains are used in a grinding wheel, different sieving methods are used to differentiate between specific grain sizes and grain shapes.

In the following only CBN grains are taken into consideration as the crystalline structure of CBN grain types allows to differentiate between significant grain shapes like shown in Figure 2. ABN is a trade mark for CBN grain material produced by the company Element Six. The crystal morphology of CBN grains can be described in a range from pure cubic to octahedral and tetrahedral forms. In the literature a big range of different CBN grain shapes is described. Geometrical bodies like spheres, ellipsoids, hexahedrons, octahedrons and dodecahedrons are used to describe the shapes of the grains [7], [9].
In grinding wheels the grains are bonded with different bond types like vitrified, resin or metallic bonding systems. Every bonding system has its specific properties as well. The porosity of the grinding wheel can be adjusted by the volumetric composition of the grain and bond material as well as pore forming material. All influences shown above define the structure of the grinding wheel and thus in combination with the dressing process the wheel topography. The topography of the grinding wheel is in contact with the workpiece and defines the performance of the grinding process. Numerous approaches to model the grinding wheel surface has been done in the past [4], [7]. An approximation of the topography and the process result can be achieved for specific grinding wheels, provided time consuming measurements of the wheel surface could have been done. The design of grinding wheels, based on the composition, has not been realized yet.

In the past several approaches to model structures like those in a grinding wheel were published. At the INSTITUTE NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA) researchers developed methods to model granular structures [2]. Their main ansatz is to solve a packing sphere problem which consists in filling a domain by balls whose radii follow a given size distribution. They solved this problem with an advanced-front approach and then used these balls as approximations to grains. Then they placed the grains inside these balls to get a distribution of grains in the domain as a model for granular structures. Researchers at the School of Mechanical Engineering & Automation at the Northeastern University Shenyang developed a model of a virtual grinding wheel and a grinding simulation in which they used an ortho-hexahedron as a basic shape for grains and randomly distribute these shapes on the wheel base [8]. They applied their model of a virtual grinding wheel to a simulation of a surface grinding process with different machining parameters. Right now these methods are efficient only in the 2D case.

2. Motivation

Today the design of grinding wheels and the composition of grains, bond material and pores is a knowledge based process which is done by experienced staff at grinding wheel manufactures. The structure of the grinding wheel together with the dressing operation defines the grinding wheel topography, which is in contact with the workpiece. In combination with the process kinematic the topography defines the performance of the grinding wheel. If the grinding wheel structure and the resulting grinding wheel topography can be modeled, the grinding wheel can be designed to meet the process requirements without using time and cost consuming trial and error methods. Hence a model to predict the grinding wheel structure based on the volumetric composition of the components would enable grinding wheel manufacturers to predict the workpiece quality and the process productivity. Also a better understanding for the complex grinding wheel can be established.
3. Single Grain Analysis

To validate the modeled grinding wheel structure, a comparison between real grinding wheel structures and the modeled counterpart need to be done. First the analysis of the single grains is shown. Currently only 2D methods are used in industrial applications to describe the shape of the grain material. To describe the shape in 3D new methods were tested. This data is used as input parameter for the grain shape distribution in the grinding wheel. In the next step, the structure and the distances between grains in the grinding wheels will be analyzed by using image processing and statistical methods.

3.1. Statistical Analysis

The properties of the grain material are essential for the chip formation in grinding processes. Hence mathematical descriptions of the grains and the statistical distribution of the different grain shapes are required.

Figure 3: SEM image of ABN300 grains (average grain size: 91 µm) and characteristic factors.

Figure 3 shows the scanning electron microscope image (SEM) of a CBN grain specimen with a grain size of 91 µm. The irregular shaped grains are strewn on a soft ground material randomly. A variety of characteristic factors can be determined via transmitted light microscopy [10]. The most common values for abrasives are the shape factor, the ellipticity, the ferret ratio and the convexity. These factors can be used to approximate the grain collective roughly. The disadvantage of this method is that the shape of particles is pictured in only one plane. Accordingly, changes in the contour of a particle in another plane cannot be detected [10]. For this reason, new methods are applied to generate valid data to evaluate the shape of the grain material in order to use it for the model input.

3.2. CT Scan Data Analysis

An approach to analyze the grain geometry is to use a Computer Tomography CT Scan of a prepared specimen containing only resin and two layers of grains. Researchers of the Institute of Plastics Processing (IKV) at the RWTH
Aachen University have done a CT Scan of the specimen resulting in about 5000 TIFF files. Each file containing $4000 \times 4000$ pixels with 16 Bit gray scale values represents a slice of the specimen. From these values we want to locate the grains in the specimen and extract the geometric shape as a triangle mesh using the Marching Cubes (MC) [5] algorithm.

We think of the data as 3D Cartesian grid in which the nodes holding gray scale values. To locate and separate the grains we prescribe an interval $U$ and apply a depth first search (DFS) to those grid nodes which values belongs to $U$ to get all connected components of the specimen. We call them feasible nodes. Because of the large size of the data we cannot compute the DFS on the feasible nodes directly. Instead we have to apply the DFS on the feasible nodes for each slice separately and then combine the information to get the connected components of the whole specimen.

Let $l, m, n \in \mathbb{N}$, $I = \{1, \ldots, l\}$, $J = \{1, \ldots, m\}$, $K = \{1, \ldots, n\}$ and $S_i \in \mathbb{N}^{m \times n}$ representing the feasible set of slice number $i$ for $i \in I$. Then for every $S_i$ we have an index set $I_i \subset \mathbb{N}$ and coordinate sets $C_i, r \subset J \times K$ representing the connected components (labeled with index $r$) of $S_i$ for $r \in I_i$ and $i \in I$, see Figure 4.

Now let $B_{i,r} \in \mathbb{N}^{m \times n}$ be defined as

$$B_{i,r} := \{ s \in I_i-1 | \exists (u,v) \in C_{i,r} \cap C_{i-1,s} \}$$

and with $\tilde{S}_i \in \mathbb{N}^{m \times n}$ defined as

$$\tilde{S}_i(u,v) := \begin{cases} 
    s & \exists s \in I_i \text{ with } (u,v) \in C_{i,s} \text{ and } \forall (u,v) \in J \times K \\
    0 & \text{otherwise}
\end{cases}$$

we have

$$B_{i,r} = \{ s \in I_i-1 | \exists (u,v) \in C_{i,r} \text{ with } \tilde{S}_{i-1}(u,v) = s \}$$

and $B_{1,r} = \emptyset$ for $r \in I_i$ and $i \in I \backslash \{1\}$.

Define $G := (V,E)$ through $V := \{(i,r) \in \mathbb{N}^2 | \exists i \in I \text{ with } r \in I_i\}$ and $E := \{(i,r),(j,s)\} \in \mathbb{N}^2 \times \mathbb{N}^2 | \exists (i,j) \in I \times I \text{ with } j = i - 1, r \in I_i, s \in B_{i,r}\}$. 

Figure 4: Two slices with connected components.

Figure 5: Triangle mesh (simplified) of an extracted grain using MC.
Now let us consider $G$ as an undirected graph, then it follows that the connected components of $G$ are in a one to one correspondence to those of our specimen. We suggest the following algorithm to compute the connected components of our specimen:

**Algorithm 1**

| Input: slices of the specimen, $l, m, n, I, J, K$; |
| 1. $S_0 \leftarrow$ compute $S_1$; |
| 2. $\forall r \in I_1$: |
| $\quad$ compute $C_{1,r}$; $% S_0 = S_1$ |
| $\quad$ save $C_{1,r}$ to a file; |
| $\quad$ save node $(1, r)$ in $G$; |
| 3. $S_0 \leftarrow$ compute $S_1$; |
| 4. $\forall i \in I$ with $i > 1$ : |
| $\quad$ $S_i \leftarrow S_i$; |
| $\quad$ $\forall r \in I_i$: |
| $\quad\quad$ compute $C_{i,r}$; $% S_1 = S_i$ |
| $\quad\quad$ save $C_{i,r}$ to a file; |
| $\quad\quad$ save node $(i, r)$ in $G$; |
| $\quad\quad$ $\tilde{S}_1 \leftarrow$ compute $S_1$; |
| $\quad\quad$ $\forall r \in I_i$: |
| $\quad\quad\quad$ compute $B_{i,r}$; $% \tilde{S}_0 = \tilde{S}_{i-1}$ |
| $\quad\quad\quad$ $\forall s \in B_{i,r}$: |
| $\quad\quad\quad\quad$ save edge $\{(i, r), (i - 1, s)\}$ in $G$; |
| $\quad\quad\quad$ swap $S_1, S_0$ and $\tilde{S}_1, \tilde{S}_0$; |
| 5. compute connected components of $G$; |
| 6. reconstruct connected components of the specimen; |
| Output: connected components of the specimen; |

Now for every connected component of our specimen we can define an own 3D Cartesian grid which nodes values are set to one if they belong to the connected component otherwise they are set to zero. In this situation we are ready to apply the MC algorithm for every connected component getting the geometric shape for every grain as a triangle mesh. See Figure 5 for an example of an extracted grain.

### 4. The Volumetric Structure Element

The goal of the development of a mathematical model is a prediction of different kinds of important properties of a grinding wheel which ensure an optimal use in the grinding process. To achieve this goal we decided to split our investigation in two stages:

1. Measuring and modeling of geometrical properties.
2. Measuring and modeling of material properties.

Here we focus only on a small part of a grinding wheel such like a cube consisting many grains. We will call it a *volumetric structure element* (VSE). In our first
approach we just model the geometrical shapes and positions of the grains in the VSE without bond material and without any pores. So in this paper we will focus only on geometrical properties such as shapes of the grains and the distribution of positions in a VSE.

4.1. Geometric Shapes of Grains

In a first step we decided to use convex polyhedrons as a an appropriate model for the grain in the VSE as seen in the state of the art. The next step will be to use convex hulls of extracted geometric shapes from the CT Scan. The chosen way to describe convex polyhedrons is to prescribe points and compute the convex hull of them. In view of manufacturers description of their idealized grain geometries (see Figure 2) we focus on basic shapes like platonic polyhedrons and modify them (see Figure 6). We have done this in the following way:

Algorithm 2

1. choose platonic polyhedron given in spherical coordinates ;
2. randomly modify angles and lengths of the vertices ;
3. randomly vary normals of planes for cutting vertices and edges ;

By the observation that computing half space intersections and convex hulls are dual to each other, we use the well known quickhull [1] convex hull algorithm to compute the geometrical shapes of the grains. Note that in the shown algorithms randomly means choosing a statistical distribution and computing elements of it.

4.2. Distribution of Grain Positions

The basic idea to get a proper VSE is to start with an initial configuration of grain positions and then iterate a process of moving grains collision free in the VSE. There is an important requirement to fulfill here. The measurements of the portion of grain material in a grinding wheel shows that it is about 42% of the whole volume. To fulfill this we suggest using sphere packings and to place grain in every sphere. It is well known that with sphere packings one can fill space with a density of $\frac{\pi}{3\sqrt{2}} \approx 74\%$. So we choose those grains which volumes are big enough only. See Figure 7 for an example.
First compute initial shapes and positions:

**Algorithm 3**

1. choose global sphere radius for all spheres;
2. choose geometric shapes and orientations of the grains;
3. compute volume of the grains via Gauss Theorem;
4. if the sum of the volume of all grains is too big:
   - enlarge the global sphere radius until the volume fits;
5. fix boundary grains;

Second iterate the following scheme for every inner grain:

**Algorithm 4**

1. randomly choose a direction;
2. compute the collision distance to all neighbor grains;
3. randomly choose moving distance smaller than collision distance;
4. move the grain;

### 4.3. Collision Distance

Let $A, B \subset \mathbb{R}^3$ be non-empty convex polyhedrons and $d \in \mathbb{R}^3$ with $||d||_2 = 1$ a given moving direction. Here we want to move $A$ along $d$. Then let $A_v, A_e, A_f$ and $B_v, B_e, B_f$ be the vertices, edges and faces of $A$ and $B$. The moving direction $d$ induces a projection plane $P = \{x \in \mathbb{R}^3 \mid x^T d = 0\}$. A brute force algorithm computes the collision distances between all combinations of faces and takes their minimum value. We want to improve this algorithm with three basic observations:

1. The convex polyhedrons are seperated into front and back faces.
2. The decision of colliding faces can be done in the projection plane $P$.
3. It is sufficient to compute distances between edges, faces and vertices.

Note that the sign of the scalar product of the normal of the faces and the moving direction separates the faces. Due to a change of basis we may assume that $d = (0, 0, 1)^T$ is given. So we only have to consider the front faces of $A$ and the back faces of $B$. We denote by $\bar{A}_v, \bar{A}_e, \bar{A}_f$ and $\bar{B}_v, \bar{B}_e, \bar{B}_f$ the projected vertices, edges and faces corresponding to the front faces of $A$ and the back faces of $B$. Using the coordinates in the projection plane we can compute the intersection rectangle $R$ of the bounding rectangles $R_A, R_B$ of $\bar{A}_v$ and $\bar{B}_v$ to reduce the number of computations. Therefore we apply algorithm 5.
Algorithm 5

Input: \( A_v, A_e, A_f, B_v, B_e, B_f, d \);
1. compute and apply change of basis ;
2. compute \( \tilde{A}_v, \tilde{A}_e, \tilde{A}_f, \tilde{B}_v, \tilde{B}_e, \tilde{B}_f ; \)
3. compute \( \tilde{R}_A, \tilde{R}_B ; \) % bounding rectangles
4. compute \( R = \tilde{R}_A \cap \tilde{R}_B ; \)
5. if \( R \neq \emptyset : \)
   \( \forall (\tilde{A}, \tilde{b}) \in \tilde{A}_f \times \tilde{B}_v : \) % triangles ↔ vertices
   compute distance of corresponding \( (A, b) \in A_f \times B_v ; \)
   \( \forall (\tilde{a}, B) \in \tilde{A}_v \times \tilde{B}_f : \) % vertices ↔ triangles
   compute distance of corresponding \( (a, B) \in A_v \times B_f ; \)
   \( \forall (\tilde{g}, \tilde{h}) \in \tilde{A}_e \times \tilde{B}_e : \) % edges ↔ edges
   compute distance of corresponding \( (g, h) \in A_e \times B_e ; \)
Output: minimum collision distance ;

5. Conclusion and Future Work

In this paper we have presented a basic model for grain geometries and their distribution in grinding wheels. Our main goal is to develop a mathematical model which has the main properties of a real grinding wheel. For the grinding process the main aspect is to know how many grains are in contact with the material and what kind of geometry they have. A good model will give a realistic prediction of the grinding wheel surface. From the CT Scan we know, that the geometric shape of a grain may not be convex, which needs to be considered. We also realized that our measurements have to be improved. Probably a concept developed in [3] could improve the quality of the CT scan data significantly. We also have to include a decimation algorithm to reduce the number of triangles per grain to reduce computation time. We have to extend our model by the bonding material and the pores to generate a realistic image of the whole structure. Also a key part of a grinding wheel simulation is to model material properties. The prediction of outbreaking grains in the grinding process is essential because it modifies the grinding wheel surface and so the surface quality of the workpiece. There is also a need to have characteristic values of the grinding wheel model which can be easily compared. Further work comparing the mathematical model with real grinding wheels has to be done.

6. Acknowledgments

The authors would like to thank the German Research Foundation (Deutsche Forschungsgemeinschaft - DFG) for the support of the depicted research within the project KL500/80-1 "Mathematical modeling of Grinding Wheel Structures". We want to thank Christoph Mülder a member of the Institute of Plastics Processing (IKV) at the RWTH Aachen University for his work on the CT Scan images and for his useful discussion about the geometry extraction.
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