JCAG - DYNAMICAL GEOMETRY AND EDUCATION SOFTWARE FOR TEACHING AND LEARNING GEOMETRY

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ABSTRACT: This paper deals with the use of Dynamic Geometry Software for teaching. We will focus on the software package ***jCAG***. This has been especially designed for teaching and learning (Descriptive) Geometry. It aids the visual understanding of essential aspects and interrelations for different phenomena. Continuously modifying the viewpoint the drawings get into motion, which clarifies the shapes of the projected 3d objects. This is the key for spatial geometric cognition. Furthermore, a simple drawing can only illustrate a special situation of a circumstance, for instance Thales' Theorem. The universal validity can be recognised by changing the parameters in such a manner, that all situations appear like in a film. The changes can be done in two different ways: Dragging with the mouse or running predefined modifications in a special presentation mode. The latter one is used in Aachen as add-on for teaching Descriptive Geometry. ***jCAG*** – including all lessons – can be downloaded from the authors homepage. Furthermore special lessons can be called directly from the Web.

Keywords: Teaching, eLearning, Descriptive Geometry, Dynamic Geometry, Vector Graphics

1. INTRODUCTION

For a long period geometry teaching has been conducted with a minimal amount of equipment – such as ruler and compasses. Nowadays the abilities of computers and software have a significant influence on teaching and learning geometry. Computer software, in particular that known as Dynamic Geometry Software, has the potential to make significant improvements in how geometry is learnt and taught. Our view is that teachers should now have at their disposal an appropriate variety of equipment from which to select. As with any approach to teaching, the educational use of information and communication technology as well as computer software needs to be well thought through and carefully planned. Students should know about the basic concepts and constructions in geometry. For this reason we have designed a special software package named ***jCAG***. Figure 1 shows a typical example for a construction from Descriptive Geometry. ***jCAG*** has a lot of features specially designed for educational purposes. Both, simple constructions and complicated dependencies can easily be animated. This has a strong influence on training spatial sense. The modifications of the projection and the scaling are the keys for seeing and thinking 3d. The changes can be done in two different ways: Dragging with the mouse or running pre-

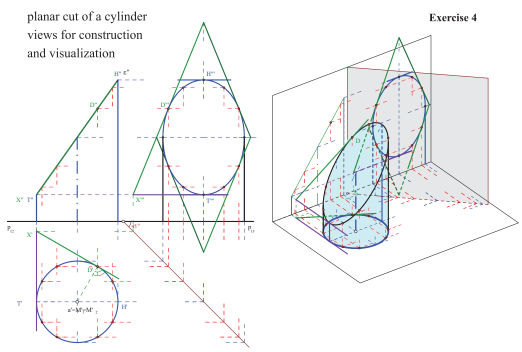

Figure 1: Planar cut of a cylinder .

defined modifications in a special presentation mode. ***jCAG*** can be used in several ways. In the classroom it can directly be called from a computer aided presentation (PowerPoint or PDF) for animating the discussed construction. In Figures 1 and 2 two typical snapshots from that series are shown. The students can invoke the same (or other) lectures in two ways. They can download and unpack – no installation necessary – the software on their computer. It is written in Java and thus independent from the operating system. ***jCAG*** only needs an installed Java Runtime Environment. Executing the program it recursively scans the demos-subdirectory and displays all available animations in a menu. Adding further animations to the demos-subdirectory they will be displayed on the next call. Alternatively, it is possible to select a special lecture-animation from our server. For every lecture there are several animations available. These can be chosen via a characteristic snapshot for that lecture, e.g. Figures 3 and 4. Clicking the picture a Java Web start is invoked. The Java Network Launching Protocol (JNLP) is used for this purpose. The basic elements of the system are points, lines, circles, conics and splines. The construction commands for object generation can be divided into five classes: Definitions, intersections, tangents, operations like midpoint, parallel line, perpendicular line etc. and handling of special objects and features. Definitions for instance are point by its coordinates or line passing through two points. The program has no restrictions on tangents and intersections. Whenever they exist it will compute them. ***jCAG*** internally stores the parametric dependencies with some extra information. This makes modifications possible (using the mouse by dragging objects). While moving objects the new drawings for each position are consequently displayed. All geometric constructions are designed to work in all meaningful situations. The right solution is chosen by the concept of continuous changes and/or with the help of the extra information. Furthermore some special commands regarding Descriptive Geometry have been implemented. For instance there is a module to get axonometric projections from two-plane projections. The layer technique enables the teacher to visualize very complex, preconstructed circumstances step by step not only in pictures but in

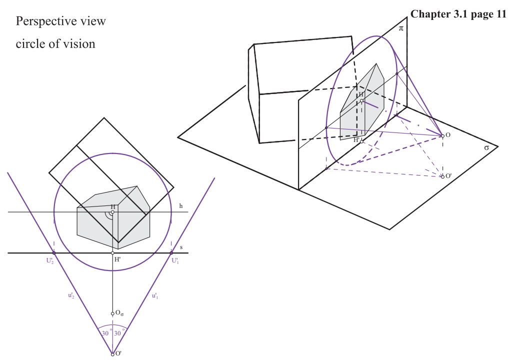

Figure 2: Perspective view of a simple building.

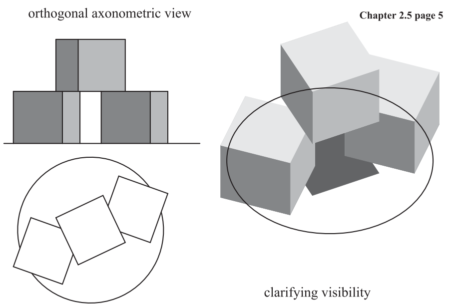

Figure 3: Further examples.

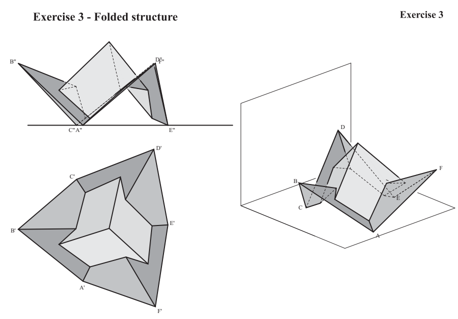

Figure 4: Further examples.

motion by a few mouse clicks. This has a strong influence on training spatial sense.

Again we refer to the Figures 1 to 4 that show advanced stages of two examples. In all snapshots an axonometric projection for a better understanding of 3d background is additionally displayed. During the lecture this is animated, too. The students can clear their backlogs by examining more examples at home in the same way. This is accomplished by demonstration-description-files. They hold the whole information for the animation - selection of layers, movement of objects etc. These are some typical examples that distinguish ***jCAG*** from other Dynamic Geometry programs. A complete set of demos for the course Descriptive Geometry for Architects has been developed.

The drawings can be exported in well suited formats for further use in publications or on web pages. Beside lossless highly compressed bitmap-files, vector graphics in various formats are supported, for instance Encapsulated Postscript (EPS), PDF and Scalable Vector Graphics (SVG). EPS and PDF files are for use in printed publications. SVGs can be used for websites. The major advantage of all the latter three formats is the very small size of the files and the possibility to zoom without grid pattern effects. This is achieved via highly accurate numerical approximation of the given geometrical entities using objects supported in the target format. The above included examples have a filesize of less than 50kB for a relative precision of 10^{-6}. This ensures the usability on websites for tablets and smartphones.

The rest of the paper is organized as follows: To get an impression of the capabilities of ***jCAG*** some typical sequences from the course in Descriptive Geometry of the architecture program are shown and explained.

Next a brief history of development stages and the use of ***jCAG*** for teaching is given. Then we exemplify the main concepts for generating illustrations with automated modifications.

2. USING *JCAG* IN THE BACHELOR OF ARCHITECTURE PROGRAM

At RWTH Aachen ***jCAG*** is used in a version with presentation modus for teaching Descriptive Geometry for students of architecture. In the following the application of ***jCAG*** its potential for didactical support is illustrated.

2.1 Goal of the course in Descriptive Geometry

The main goal of the course in Descriptive Geometry (DG) is to train student's spatial imagination and thinking. As these abilities are not adequately taught in German schools, certain efforts are required to prepare beginning students for studying at an academic level. Freshmen's competences in spatial thinking can be best supported by the abstractions used in DG combined with manual drawing. This learning is supported by the use of animated drawings excellently. As a side effect students acquire reliable handling of basic drawing technics.

2.2 Teaching conditions for Descriptive Geometry at RWTH Aachen

DG courses for RWTH's students of architecture carry a teaching load of merely two hours a week during the first year. About 250 students take DG lectures each year. In order to lift students' competence to a high level under these unfavourable teaching conditions a complex teaching system was developed and used in practice. One feature of this system is the precisely balanced employment of manual drawing and computer-aided animated visualizations.

2.3 Using *jCAG* in DG courses

In order to present as much input as possible during the lectures and to reach a high learning effect a special collection of worksheets was prepared (see [4]). The worksheets include supporting drawings (ground-plan, elevation and illustrating axonometric) introductory and explaining texts, descriptions of constructions as well as drawings or photographs of real life objects. In the lecture the instructor explains the construction by

Figure 5: Use of the worksheets and ***jCAG*** by instructor and students

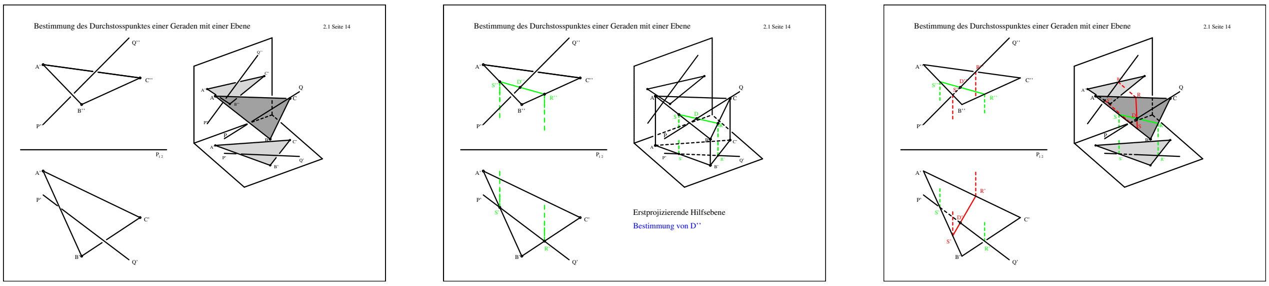

Figure 6: Determining the intersection of a line (ray of light) and a plane

complementing the worksheet sketches using different colors.

The students simultaneously transfer these additions to their own copy of worksheets. By this method students begin to critically reflect the subject already during the lecture while at the same time maintain their attention. Throughout the construction the use of different colors emphasizes reoccurring elements of construction wherever possible. As a consequence specific elements of construction can be identified easily within complex drawings and corresponding explanations of the geometrical background can be related in the worksheets. Using of the Dynamic Geometry program ***jCAG*** on a beamer in a lecture room therefore supports students' drawing by their own hand (see Figure 5). The drawing, which students and teacher work on jointly, is varied dynamically enabling a more detailed illustration of geometrical phenomena. Due to the variety of options offered by ***jCAG*** the lecturer has to choose his favourable geometrical movements beforehand. Once everything is prepared beforehand using ***jCAG*** the instructor is able to concentrate on his explanations to the students during the lecture. In the lecture room the selected movements are executed by "blind" mouse clicks. This way ***jCAG*** can precisely visually support the understanding of the important aspects of different constructions. In the next section several modifications of parameters in the drawings are illustrated with the example of the construction of shadows.

2.4 Modifying parameters using the example of shadows

The display of light and shadows is a very useful technic to improve the three-dimensional impression of architectural drawings. Ground plans and views appear more realistic when protrusions and recesses can be clearly identified. Even in axonometric and perspective the three-dimensional impression can be improved by adding shadows. As even simple tasks in Monge Projection need a sound understanding of the spatial context, the construction of shadows is an ideal tool for build-

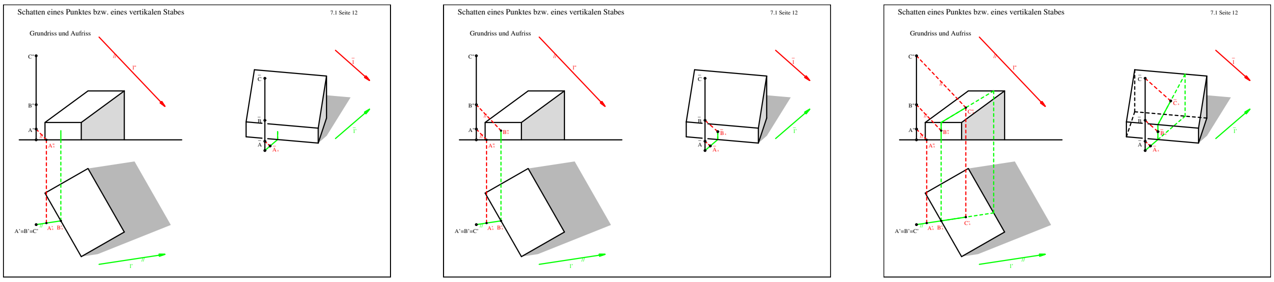

Figure 7: Finding the penetration point of a line (beam of light) in a plane

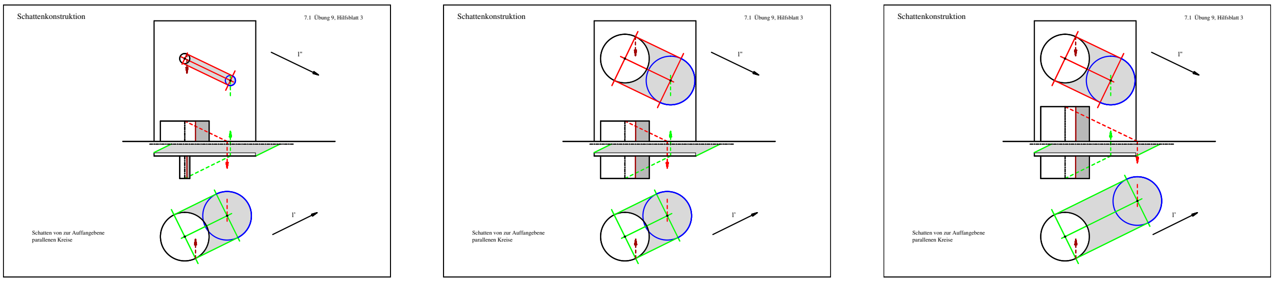

Figure 8: Shadow of straight cylinders in ground plan and elevation

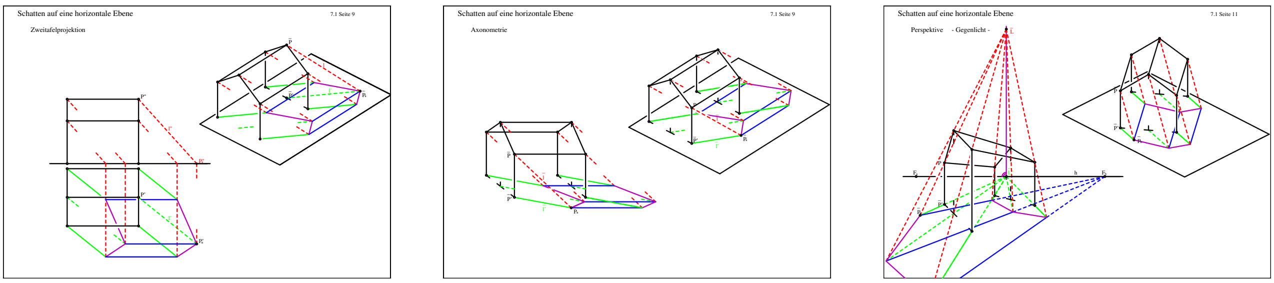

Figure 9: Shadow of a framework onto a horizontal plane

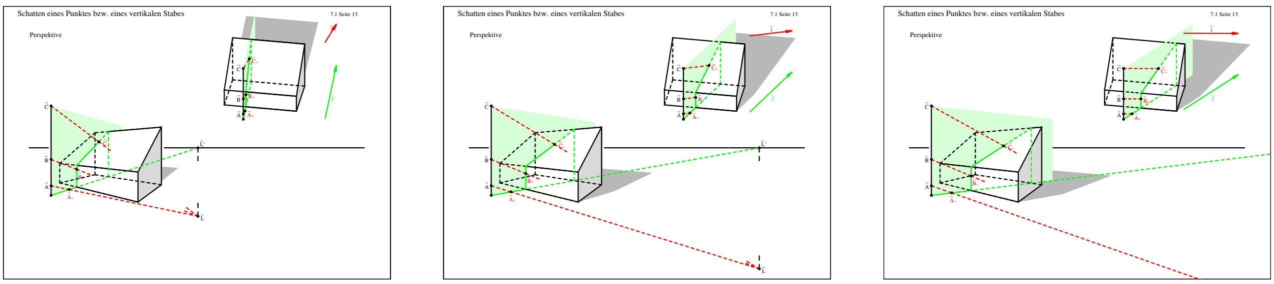

Figure 10: Shadow of a pole onto a horizontal, vertical and inclined plane (plane of light)

ing up spatial competence. Furthermore this topic perfectly consolidates the knowledge of the projection. ***jCAG*** allows to display drawings in motion, thus supporting the understanding of the construction of shadows.

a) Step-by-step development of a drawing

jCAG allows to visualize the step-by-step process of drawing. Different constructions shown in the worksheets can be retraced step-by-step (Figures 6 and 7). In particular the understanding of complex drawings is facilitated when their sequence of construction steps can be replayed as often as needed.

b) Modifying of object parameters

The geometric nature of objects can be illustrated by modifying their parameters interactively. In Figure 8 the radius and the height of a straight cylinder are changed at the same time in ground plan and elevation.

c) Simultaneous visualisation of different ways of projection

The reduction of space to a drawing plane makes great demands on spatial thinking due to its level of abstraction. This is especially true for first-year students. The comprehension of Monge projection is facilitated when it is displayed next to a corresponding axonometric. ***jCAG*** offers this kind of presentation. In Figure 9 different kinds of projections are shown next to an axonometric.

d) Modifying the position of objects in space

The understanding of a spacial situation shown in Monge projection is facilitated by the option to modify the position of the objects in space. With ***jCAG*** this modification can be simultaneously observed in ground plan, elevation and axonometric. In Figure 10 the direction of the beams of light and thereby the plane of light generated by a vertical pole are rotated.

e) Modifying the parameters of a projection

For architects the choice of the direction of view is crucial for plan presentation. In the example, the direction of view is modified in a parallel projection onto a horizontal image plane, the inclination of the projection beams staying at 45°.The position of the beam of projection is shown in ground plan. Lines of construction for the axonometric can be interpreted as rays of light projecting the edges of the building onto the ground plane (see Figure 11).

f) Dynamic modification of the direction of view in a presentation drawing

A very concise spatial effect is achieved by modifying the direction of view in a presentation drawing. The dynamics of the movement adds time as a third dimension to the 2D-drawing, thus creating a 3D-impression of the shown situation (compare Figure 12).

g) Modifying the type of illumination

Using ***jCAG*** it can be illustrated in an ideal way that parallel illumination is a special case of central illumination, as the source of light is moved into infinity (compare Figure 13).

h) Visualization of complex contexts

jCAG is also a very powerful tool to visualize complex contexts. In Figure 14 the options for illumination in perspectives are shown.

3. *JCAG* – A SHORT OVERVIEW OF THE DEVELOPMENT PHASES

The development of ***jCAG*** startet in 1985 with a DOS version called CAG (**C**omputer **A**ided **G**eometry). The origin was based on the need of an efficient tool for generating and modifying drawings for the education in Descriptive Geometry. Already at that time several systems for 2d and 3d CAD exist. Most of them only stored the objects or / and the information of the drawn parts. Thus at the moment of construction the generating commands are available but due to the fact that they are not stored modifications depending on the variation of the generating objects were not possible. The necessary construction command was missing at that stage. This was the starting point of the system CAG.

Already in 1987 a powerful interactive version was available. The basic objects were points,

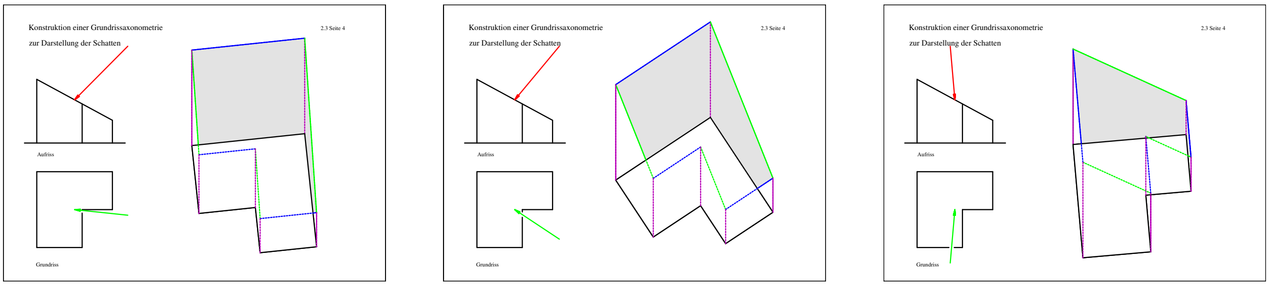

Figure 11: Shadow onto the ground plane

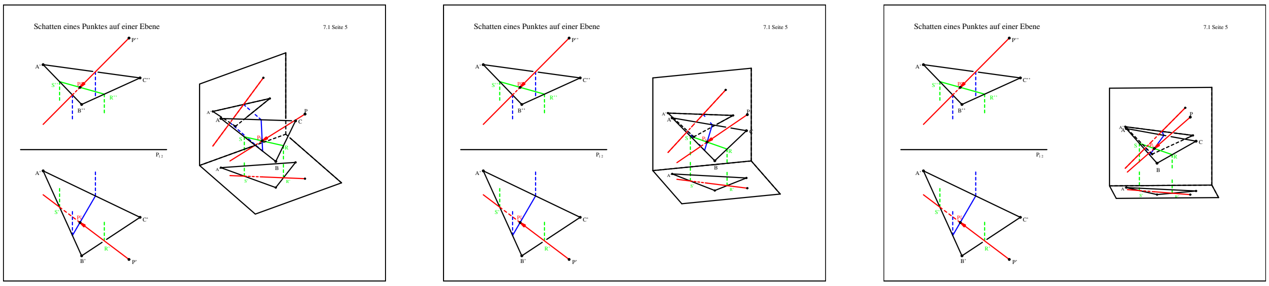

Figure 12: Shadow of a point onto a plane

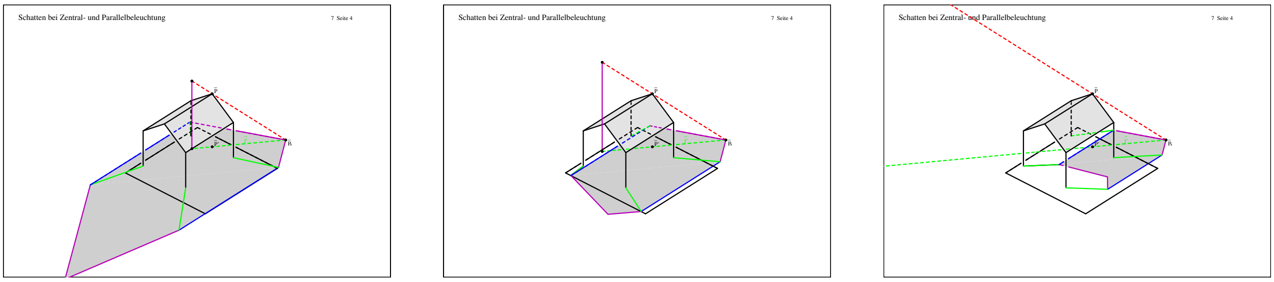

Figure 13: Central and parallel illumination

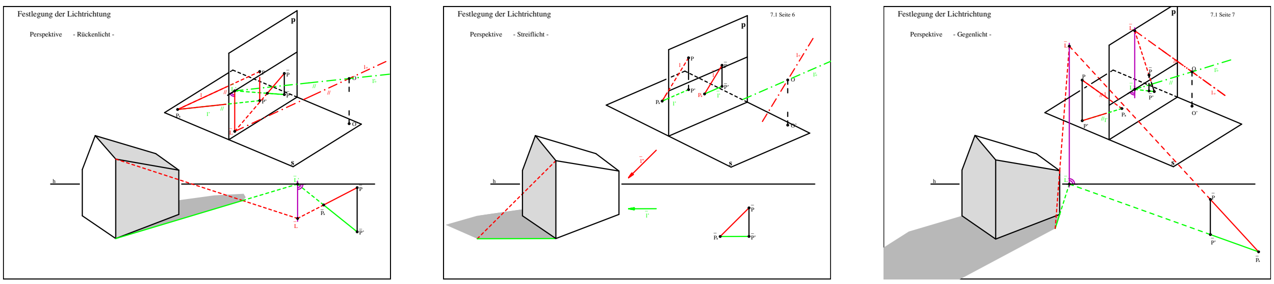

Figure 14: Shadow - light from behind, from side and from front

lines, circles, ellipses and splines. Operations like midpoint, perpendicular bisector, etc. towards all possible intersections and tangents where implemented. While the user does the construction the system stores the corresponding commands along with some optional extra information for uniqueness of the solutions with ambiguous operations. Modifications of the initial objects are consequentially applied on the constructive commands of the following objects. For this reason it is possible to modify a drawing in such a way that it fills the paper, the parts of different constructions are feasible and do not overlap (lucidity) and intermediate objects of the design fit to the page if needed.

In addition the system could store and easily apply macros only asking for some extra information during the construction. Due to this fact complicated construction steps can be repeated very quickly even for situations in which the intermediate objects did not fit onto the page, e.g. inclined cutting plane method. The system was used by a draftsman, scientific assistants and student assistants for preparing lectures and exercises. The primary objective was fulfilled and a brief documentation of that phase can be found in [1].

In the winter semester 1987/88 we started using the system for education with students of mechanical and civil engineering. The robustness of the systems was enhanced for this purpose and the program surface was reorganized. In principle the development was finished at the end of 1987. For nearly ten years the code was only maintained. It should be mentioned that low computational power of the computers in that period (6MHz ATs with 640kB memory) the changes were only delivered on call and not *on the fly*. In [2] the DOS version is explained in detail.

In the mid 90^{th} the performance of the PCs was significantly increased. Now the new aim was to have a system that permanently recomputes and redisplays the actual scene due to the modifications made by dragging objects with the mouse. For this reason CAG which was implemented in standard Turbo Pascal for DOS with the addition of assembly code for efficient drawing was ported to Windows. The possibilities for modifications were enhanced. Especially mentionable is the module for automatically running predefined modifications for complex constructions. To simplify the generation of spatial illustrations a *pseudo 3d* module was integrated into the system. To handle substantial drawings and its details improved layer techniques were introduced. In this form we started using it as add-on for teaching Descriptive Geometry at RWTH Aachen University. At that stage the drawings could be exported as compressed bitmap (BMP with run length encoding) or as vector graphics (Encapsulated Postscript) for use in publications. Again the development was stopped for a long period.

Recently we decided to totaly redesign the code and port it to Java making it independent from the operation system. Furthermore we want to have a version running on a web server. We named this version ***jCAG***. Still no installation is necessary. It is sufficient to download and unpack it. ***jCAG*** only needs an installed Java Runtime Environment. Alternatively, a lot of special lecture-animations can directly be called from our server. These can be chosen by clicking a characteristic snapshot for it. A Java Web start is invoked using the Java Network Launching Protocol (JNLP) for this purpose.

4. AUTOMATED ANIMATIONS

To clarify the concept of modification we exemplarily take a look at the points. Regarding changes we distinguish between three kinds of points

1. Points with two degrees of freedom

2. Points with one degree of freedom

3. Fixed points

Incidences can be modified afterwards. During the construction ***jCAG*** automatically detects most of them, for instance point on a circle or intersection between two conics. Although points of the third kind can be modified, e.g. redefinition of intersections, they are not suitable for the design of demos. We need at least one degree of freedom to get the pictures into motion. The command for generating a point is internally stored in a (simplified) form as

- P=DefinePoint(x, y**)**

where x and y are the (2d) coordinates of **P**. It can be dragged with the mouse or one can give new coordinates by the keyboard. But there is another more interesting way for automatic and predefined modification. The syntax of that command is

- ModifyPoint(P; x_0, x_1, dx; y_0, y_1, dy**).**

The point **P** will then bounce in a rectangle given

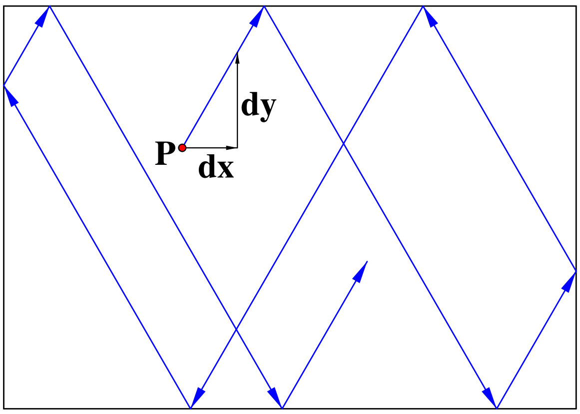

Figure 15: Bouncing of points.

by $[x_0, x_1] \times [y_0, y_1]$, stepping in x-direction with step size dx and in y-direction with step size dy. At the borders of the rectangle it bounces due to the law "angle of reflection equal to angle of incidence". Figure 15 shows an example for the above mentioned automatic modification. In Figure 2 the command can be used to automatically change the point of view. Similar commands exist for change lines, e.g. the rotation angle, circles, e.g. the radius, ellipses, e.g. center and principal axis, and splines.

5. THE PROJECTION MODULE

A powerful tool for spatial demonstration is the projection module of ***jCAG***. The special arrangement of an orthographic view (top and front view) can be used to induce a coordinate system (see Figure 16). This is simply done by defining the

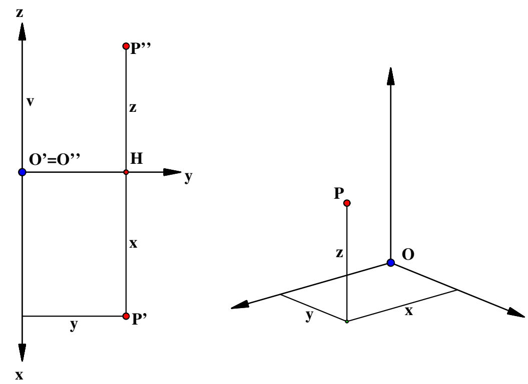

Figure 16: Projection module.

point $\mathbf{O}' = \mathbf{O}''$. The axes are then assumed to be vertical and horizontal respectively. The projections of a point P in top and front view are $\mathbf{P}'$ and $\mathbf{P}''$. From these two projections we can compute the coordinates as shown in figure 16. Given an arbitrary mapping and a position **O** ***jCAG*** is able to compute the image **P**. All coordinates of **P** (x, y and z) and the parameters of the projection can be used for automatic modifications. With the notations of Figure 16 we can state the essential commands in the following form

- **P′=DefinePointRelative(O′**, x, y**)**
- **P″=DistanceFrom(H**, v; z**)**
- **DefineMap(O′, O**, $kind$, ϕ, θ, r, $scale$**)**
- **ModifyPoint(P″**; z_0, z_1, dz**)**
- **ModifyMap(**φ_0, φ_1, $d\varphi$; θ_0, θ_1, $d\theta$**)**

The parameters φ, θ, and r represent spherical coordinates. The parameter r (distance) is only needed for perspective views. All other parameters should be obvious. There are further commands for the automatic modification of maps.

A more complex examples is given in Figure 17. All projection parameters for the view can be changed. A simple use of some special functions from ***jCAG*** leads to Figure 18

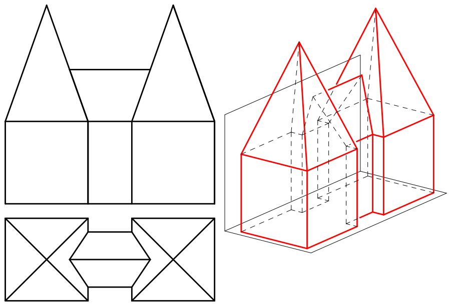

Figure 17: Projection module.

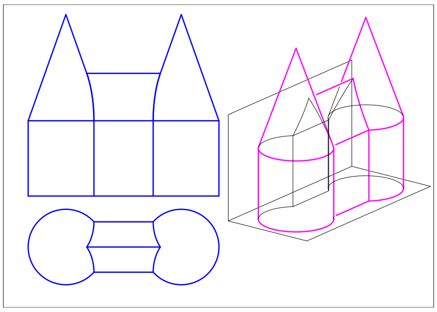

Figure 18: Modification of Figure 17

6. CONCLUSIONS

In this paper only a view aspects of ***jCAG*** and its use for improving geometrical teaching could be shown. One has to look at real animation and online presentations to get an intrinsic overview of its capabilities. This is true for any other software on Dynamic Geometry. Such packages together with well selected demos and animations can not replace the classroom teaching, but they offer a tremendous contribution for didactically aid and to spatial geometric cognition. Due to its special design for (Descriptive) Geometry ***jCAG*** is a mighty tool for significant improvements in teaching and learning geometry.

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