

We present a coupling between a pointwise particle of masse m and position $h(t)$ and an inviscid fluid having, at each time t and point x , a velocity $u(t, x)$ and a density $\rho(t, x)$. The coupling is achieved through a drag force D . Our system writes

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0 \\ \partial_t(\rho u) + \partial_x(\rho u^2 + c^2 \rho) = -\lambda D(\rho, u - h') \delta_{h(t)}(x) \\ m h''(t) = D[\rho(t, h(t)), u(t, h(t)) - h'(t)] \end{cases}$$

We suppose that D has the same sign that $u - h'$. The last line show that the particle accelerates if it travels slower that the fluid. The first line is the conservation of the fluid's mass and the second line translates the action-reaction principle. We quickly focus on the Riemann problem for the one way coupling

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0 \\ \partial_t(\rho u) + \partial_x(\rho u^2 + c^2 \rho) = -D(\rho, u) \delta_0(x) \\ \rho|_{t=0} = \rho_L \mathbf{1}_{x<0} + \rho_R \mathbf{1}_{x>0} \\ u|_{t=0} = u_L \mathbf{1}_{x<0} + u_R \mathbf{1}_{x>0} \end{cases}$$

where the particle is now motionless. This system is interesting for itself. It models, for example, the behavior of a fluid slowed down by a grid.

Those systems are easily obtained and reflect the assumptions on the interactions between the fluid and the particle, but they are not well posed. Indeed the Euler equations considered here are inviscid, so there is no reason for u and ρ to be continuous at $h(t)$. As a consequence, the right hand sides do not make sense. The aim of this presentation is to explain how to treat this type of source term. We first give a precise definition of the non conservative product $D(\rho, u) \delta_0$. Then, we give some ideas on its influence on the Riemann problem. Its structure is rather complicated, and develops up to 4 waves which can interact. Moreover, depending on the form of the drag force D , it is possible to have up to three solutions. Eventually and if time allows it, we will introduce a finite volume scheme for this problem, show some numerical simulations and explain the challenges on the numerical analysis of such a scheme.