Large Time Step Finite Volume Methods for the Shallow Water Equations

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Abstract

Multiscale phenomena appear in geophysical flows. Their simulation challenges numerical schemes and has received significant attention in literature.
In case of the shallow water equations, low Froude numbers force explicit numerical schemes to compute with very small time steps due to the CFL stability constrain and cause high computational effort. Implicit-explicit (IMEX) schemes weaken this constrain significantly. Therefore they are also called *large time step methods*.
We present new large time step methods for the shallow water flows. In order to take into account multiscale phenomena nonlinear fluxes are splitted into a linear part governing the gravitational waves and the nonlinear advection. We approximate fast linear waves implicitly in time and in space by means of the genuinely multidimensional operator. On the other hand we approximate nonlinear advection part explicitly in time and in space by means of the method of characteristics or some standard numerical flux function. Time integration is realized by IMEX methods. We apply the IMEX Euler scheme, two step Runge Kutta Cranck Nicolson scheme as well as the semi-implicit BDF scheme and prove their asymptotic preserving property with respect to low Froude number.
Numerical experiments demonstrate stability, accuracy and robustness of these new large time step finite volume schemes with respect to small low Froude numbers.
By the cost of solving a linear equation at every time step the presented schemes reduce the number of time steps clearly. Reduction of around one hundred times have been observed.

Key words: low Froude number flows, asymptotic preserving schemes, shallow water equations, large time step, semi-implicit approximation, evolution Galerkin schemes

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