Well-Balanced Positivity Preserving Central-Upwind Scheme for the Shallow Water System with Friction Terms

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In this talk, I will discuss the effects of the bottom friction terms in the shallow water model given by

$$\begin{cases} (h)_t + (hu)_x + (hv)_y = R, \\ (hu)_t + \dfrac{\partial}{\partial x}\left(hu^2 + \dfrac{g}{2}h^2\right) + \dfrac{\partial}{\partial y}(huv) = -ghB_x - \dfrac{\tau_x}{\rho}, \\ (hv)_t + \dfrac{\partial}{\partial x}(huv) + \dfrac{\partial}{\partial y}\left(hv^2 + \dfrac{g}{2}h^2\right) = -ghB_y - \dfrac{\tau_y}{\rho}, \end{cases} \tag{1}$$

where $h(x,y,t)$ is the water depth, $u(x,y,t)$ and $v(x,y,t)$ are the x- and y-components of the average velocity, $R(x,y,t)$ is the water source term, $B(x,y)$ is a function describing the bottom topography, g is the gravity constant. Functions τ_x and τ_y are the two components of the bottom friction and ρ is the water density. The friction terms are computed from the following formulae (using the classical Manning formulation):

$$\frac{\tau_x}{\rho} = \frac{gn^2}{h^{1/3}}\, u\sqrt{u^2+v^2}, \qquad \frac{\tau_y}{\rho} = \frac{gn^2}{h^{1/3}}\, v\sqrt{u^2+v^2}, \tag{2}$$

where n is the Manning coefficient.

We are interested in simulating drainage of the rain water in urban areas. In such situations, the simplest yet physically relevant quasi one-dimensional steady-state solutions correspond to the case when the water flows over a slanted infinitely long surface with a constant slope and are described by

$$h \equiv \text{constant}, \quad u \equiv \text{constant}, \quad v \equiv 0, \quad B_x \equiv \text{constant}, \quad B_y \equiv 0, \tag{3}$$

or

$$h \equiv \text{constant}, \quad u \equiv 0, \quad v \equiv \text{constant}, \quad B_x \equiv 0, \quad B_y \equiv \text{constant}. \tag{4}$$

I will present a new well-balanced central-upwind scheme that is capable of exactly preserving its steady states (3) and (4). The scheme also preserves the positivity of the water depth. The designed scheme has been tested on a number of numerical experiments including the ones with realistic urban bottom topography structures. The obtained results demonstrate a superb performance of the proposed numerical method. In particular, the data in one of the numerical examples correspond to the laboratory experiments, which designed to mimic the rain water drainage in urban areas containing houses. Since the rain water depth is typically several orders of magnitude smaller than the height of the houses, the new scheme implemented in a straightforward manner may fail to accurate reproduce the experimental results. Therefore, a special numerical technique has been developd so that the houses are removed from the computational domain, which becomes a punctured domain with many internal solid wall boundary pieces. The rain water falling over the houses is then redistributed to the areas near the edges of the houses. This helps to achieve a remarkable agreement between the numerical and experimental results.

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