Analysis of residual based discretizations of the enhanced Boussinesq equations for wave propagation over complex bathymetries

A.G. Filippini and M. Ricchiuto

Abstract

In this work we consider the solution of the enhanced Boussinesq equations of Madsen and Sørensen (Coast.Eng. 18, 1992) by means of residual based discretizations. In particular, we investigate the applicability of upwind and stabilized variants of the centred Residual Distribution and Galerkin linear finite element schemes for the simulation of wave propagation and transformation in the near-shore regions over complex bathymetries. These techniques have already been successfully applied to the solution of the nonlinear Shallow Water equations (Ricchiuto and Bollerman J.Comput.Phys 228, 2009 - Hauke CMAME 163, 1998), showing very high potential in terms of preservation of physically relevant steady equilibria (well-balancedness), and of stable approximation of moving shorelines.

In order to discretize the system of equations through a linear finite element approximation of the variables, the higher order spatial derivatives are rewritten in a lower order form by means of the introduction of auxiliary variables. The Crank-Nicholson method is then used for the integration in time (due to its semplicity, A-stability character and non-dissipative nature) coupled with a Newton's iteration cycle, to solve the nonlinear character of the obtained schemes, with frozen Jacobian in order to reduce the computational cost of the procedure.

Thourough numerical tests have been performed on solitary wave propagation ane interaction with complex bathymetries, and periodic wave dffraction in one and two space dimentions.

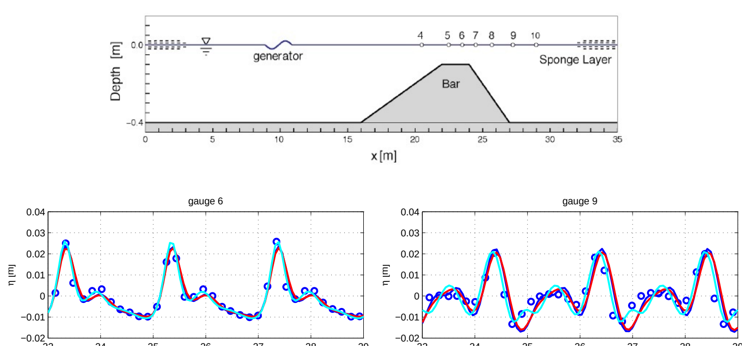

Figure 1: Results for the periodic wave propagation on a sumberger bar.

The work realized constitutes a first step toward the obtention of a model coupling the enhanced Boussinesq equations with the Shallow Water equations for the description in wave breaking regions. The contribution of the present work is to show that equal order and even low order (second) upwind/stabilized techniques can be used to model non-hydrostatic wave propagation over complex bathymetries. This claim is supported by theoretical error analyses and thorough numerical tests showing excellent agreement (fig.1) between computed and analytical solutions or experimental data.