A class of incomplete Riemann solvers based on uniform rational approximations to the absolute value function

M. J. Castro, J. M. Gallardo

Departamento de Análisis Matemático, Universidad de Málaga

castro@anamat.cie.uma.es, jmgallardo@uma.es

A. Marquina

Departamento de Matemática Aplicada, Universidad de Valencia

marquina@uv.es

Abstract

In this work we propose a new class of incomplete Riemann solvers, based on approximations in the L^∞-norm to the absolute value function in $[-1,1]$ by means of rational functions, for the numerical approximation of the solution of conservative and nonconservative hyperbolic systems. The main idea relies on the construction of a numerical approximation to the viscosity matrix by using an appropriate rational real function $R(x)$, that approximates the function $|x|$ uniformly in $[-1,1]$, evaluated at the Jacobian of the fluxes of the hyperbolic system computed at some average value (for example, Roe averages). In addition to the Jacobians of the fluxes we shall use either the maximum in absolute value of the characteristic speeds in each cell or an upper bound of them. Thus, the resulting approximate Riemann solver is incomplete in the sense that we do not use the complete spectral decomposition of the Jacobian. Moreover, the new class of Riemann solvers consists of a hierarchy of schemes running from the more dissipative to the less dissipative ones, and having as limiting case a Roe-like scheme. According to the order of the approximation of the generating rational function used, the degree of dissipation can be dosed for particular applications. We study different rational approximations: Newman-type functions, iterative generated Halley functions, and also Chebyshev polynomial approximants. We test our basic algorithms for different initial value Riemann problems for ideal gas dynamics (HD) and magnetohydrodynamics (MHD) to observe their behavior with respect to challenging scenarios in numerical simulations, including some standard numerical pathologies (e. g., heat conduction, postshock oscillations and overheating) and the formation of compound waves in ideal MHD. We also examine our proposed schemes, by computing the numerical approximation of different initial value problems for nonconservative multilayer shallow water equations, where it has been observed that intermediate waves can be properly captured for an appropriate degree of approximation of the generating rational function used. Our numerical tests indicate that the proposed schemes are robust, running stable and accurate with a satisfactory time step restriction (CFL constant), and the computational cost is more advantageous with respect to schemes that use a complete spectral decomposition of the Jacobians.