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Explicit Higher Order Schemes for Coupling Dimensionally Heterogeneous Free-Surface Flow Models

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The two-dimensional shallow water equations (2dSWE) are widely used in the simulation of shallow free-surface flows. Sometimes even one-dimensional models may be accurate enough at least in some areas of the computational domain to capture the hydrodynamics of interest, at much lower computational cost.

In order to reduce the overall computational time spent, a number of numerical methods for coupling one- and two-dimensional shallow water models have been proposed, see e.g., [2, 5]. Only very recently, adaptive model coupling has been investigated (see [4]). We assume an *a-priori* model decomposition, i.e., the computational domain is decomposed into several sub domains where either the full 2dSWE or lower-dimensional 1dSWE shall be solved.

We derive a spatially heterogeneous discontinuous Galerkin scheme of potentially arbitrary order to be carried out in the whole two-dimensional computational domain. In the *a-priori* fixed 1d flow region the method is equivalent to a one-dimensional scheme of equal order, minimizing the number of degrees of freedom [3]. We apply a generic stabilization approach in the spirit of [1] combining shock detection methods and a slope limiter. We illustrate the reliability of the scheme and the accuracy of the numerical solutions obtained in suitable test cases.

References

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