The Rankine-Hugoniot-Riemann solver for multidimensional conservation laws with source terms

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The *Rankine-Hugoniot-Riemann* (RHR) solver has been developed to solve multi-dimensional conservation laws with source terms, a scheme which was first suggested by Jenny and Müller [*J. Comput. Phys.*, 145, 1997, pp. 575–610]. The solver uses a novel way of treating the flux gradients in one dimension as source terms in the other dimensions. The total source term, which includes the ordinary source term and the cross flux gradient, is imposed as a singular source in the middle of each cell in a finite volume grid. LeVeque [*J. Comput. Phys.*, 146, 1998, pp. 346–365] imposed the source in the cell centre in a similar way, but without including cross flux gradients as source terms.

By integrating the original PDE over this singular term, one obtains a Rankine-Hugoniot-like condition that causes a jump in the solution over the singular source, given by

$$f(u_{\mathrm{east}}) - f(u_{\mathrm{west}}) = Q, \tag{1}$$

where f is the flux function, u the conserved variables, and Q the total source term. The new half-states in the left/western and right/eastern part of the cell are then used when solving the Riemann problems at the interfaces, which can be solved using an ordinary Riemann solver.

We introduce a new limiter that ensures that the half-states do not exceed the states in the neighbor cells, to resolve stability issues reported by Jenny and Müller. We prove analytically that the solver is of second order for rectangular grids for a scalar advection equation with a linear source term. Second order is also demonstrated numerically for the 2D isothermal Euler equations and the 2D shallow water equations. We compare the error of the RHR solver to that of a MUSCL scheme, and find the RHR solver to be more accurate.