## Hyperbolicity of the multi-layer shallow-water model with free surface

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## ABSTRACT

Considering n fluids, which are homogeneous, inviscid, incompressible and without surface tension, the 3D Euler equations with long wave assumptions provide the multi-layer shallow-water model with free surface. Since salinity is assumed discontinuous, these equations can be used to describe the oceans. The study of the hyperbolicity of this quasi-linear PDEs system is divided in two parts.

First, we present the hyperbolicity field of the two fluids particular case. This field is composed of two parts, as noticed numerically in [2]. Consequently, it is possible to approximate the eigenstructure, as initiated by [4] and used in [1] or [3].

Second, a system with n fluids is considered. As the PDEs has many parameters, the exact description of the hyperbolicity field is extremely complex. However, it is possible to get an approximation by relying on the two-layer case. Therefore, the eigenstructure is estimated and provides the elements needed for a Riemann solver. Comparison with numerical applications will be presented.

## References

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