# Discontinuous Galerkin Approximation for a nonlinear-diffusion Kolmogorov-Fisher's equation

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**Abstract**

We develop a new high-order numerical method for the approximation of the porous Fisher Kolomogorov's models

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{D}{\alpha+1}\Delta u^{\alpha+1} &= Ru(1-u^{\beta}) && \text{in } \Omega\times(0,+\infty), \\ \nabla u^{\alpha+1}\cdot \boldsymbol{n}_{\Omega} &= 0 && \text{on } \partial\Omega\times(0,+\infty), \\ u_{|_{\{t=0\}}} &= u_0 && \text{in } \Omega, \end{aligned} \tag{1}$$

with positive parameters $R$, $\beta > 0$, and $\alpha \geq 1$, and initial datum $u_0$ taking values in $[0,1]$. This kind of reaction-diffusion equations play an important role in dissipative dynamical systems for physical, chemical and biological phenomena. The solutions of this family of equations are characterized by propagating fronts, which can be very steep [1]. From a numerical point of view, the discretization of this problem is challenging because the numerical scheme has to reproduce shock waves or fronts of the analytical solutions, and preserve stability and invariance properties.

The proposed approximation method is based on a discontinuous Galerkin discretization in space and on an explicit Runge-Kutta scheme in time. This method can be seen as the relaxed limit of a relaxation system and extends our previous work on porous media equation [2]. Relaxation schemes take advantage of the replacement of the original partial differential equation with a semilinear hyperbolic system of equations, with a stiff source term, tuned by a relaxation parameter $\epsilon$. When $\epsilon \to 0^+$, the system relaxes onto the original PDE: in this way, a consistent discretization of the relaxation system for vanishing $\epsilon$ yields a consistent discretization of the original PDE. The numerical schemes obtained with this procedure do not require solving implicit nonlinear problems and possess the robustness of upwind discretizations.

These methods are capable to reproduce the main properties of the analytical solutions. We present some preliminary theoretical results and provide several numerical tests.

[1] T. P. Witelski, Merging traveling waves for the porous-Fisher's equation, *Appl. Math. Lett.*, vol. 8, no. 4, pp. 57-62, 1995.

[2] F. Cavalli, G. Naldi, and I. Perugia, Discontinuous Galerkin approximation of relaxation models for linear and nonlinear diffusion equations, *SIAM J. Sci. Comput.*, vol. 34, pp. A137-A160, 2012.