A bilayer model for the transport of pollutant film over water

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Abstract

The objective of this work is to deduce a model to simulate the transport of a thin film of pollutant over water. Several models have been derived in order to consider the evolution of immiscible fluids. But the difficulty in this situation lies not only in the different physical properties of the fluid but also in the different aspect ratios regarding the thickness of each layer.

The proposed model is derived from the incompressible Navier-Stokes equations together with suitable boundary conditions including friction and capillary effects. The derivation is based on the different properties of the fluids, thus, we perform a multiscale analysis in space and time, and a different asymptotic analysis to derive a system coupling two different models: the Reynolds lubrication equation for the upper layer and the shallow water model for the lower one. We also prove that the model is provided of a dissipative entropy inequality, up to a second order term.
The novelty of this work rests on two features, that, to the best of our knowledge, have not been tackled before in the modelling of multilayer systems. The first one is the coupling of two different equations on a bilayer model and the second one is the multiscale analysis −in space and time− developed in the two layers. Both of them provide also the main difficulties to derive the proposed model.

We will present some academic numerical tests where the model may be suitable of application. With these results we point out the necessity of considering a Reynolds lubrication theory to model the evolution of the thin film flow. The two different scales are also needed in order to be coherent with the physical behaviour of two immiscible fluids.

References

[1] F. Gerbeau, B. Perthame. *Derivation of viscous Saint-Venant system for laminar Shallow-Water; Numerical validation.* Disc. Cont. Dynam. Syst. Series B 1(1), 89–102, 2001.

[2] G. Narbona-Reina, J. D. D. Zabsonré, E.D. Fernández-Nieto, D. Bresch. *Derivation of a bi-layer Shallow-Water model with viscosity. Numerical validation.* CMES 43(1), 27-71, 2009.

[3] A. Oron, S. H. Davis, S. G. Bankoff. *Long scale evolution of thin films.* Rev. Mod. Phys. 69(3), 931–980, 1997.

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