Well-balanced high-order numerical schemes for one-dimensional blood flow in vessels with varying mechanical properties.

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We consider here a one-dimensional mathematical model for blood flow in thin-walled deformable elastic tubes with varying mechanical properties. The corresponding PDE system can be written as a system of balance law whose flux function depends on the spatial variable x through the mechanical properties of the tube. The goal is to develop well-balanced, high-order numerical methods for the system.

In order to achieve this goal, we use first the Generalised Hydrostatic Reconstruction technique to derive first-order numerical method that exactly solves all the stationary solutions of the system. Next, this first order method is extended to high order by using the ADER framework. Although a conventional non-linear spatial reconstruction operator is considered, some modifications are needed to preserve the well-balanced properties of the first order method.

A carefully chosen suit of test problems is used to systematically assess the proposed schemes and to demonstrate that well-balanced properties are mandatory for obtaining correct numerical solutions for both steady and time-dependent problems.

References

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