A new closure for the 5-moment equations in kinetic gas theory

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Abstract

Moment equations offer a general framework for the approximation of the Boltzmann equation. They were introduced by H. Grad in kinetic theory, who proposed to close these systems by a Hermite expansion of the distribution function in velocity space. While this approach yields closed-form expressions for the unknown fluxes, the resulting systems have limited regions of hyperbolicity, greatly limiting the usability of this closure. On the other hand, the maximum entropy approach yields hyperbolic systems of partial differential equations. Unfortunately, no closed form expressions for such moment systems including terms higher than the second moment could be found, thus requiring the usage of expensive numerical approximations. We present a new closure for modeling moderately rarefied gases, which exhibits desirable characteristics of the maximum entropy closure, such as an extended range of hyperbolicity. In contrast to the maximum entropy closure, the proposed closure is given in closed form allowing an efficient computation of the fluxes. Furthermore, while the maximum entropy closure for the 5-moment equations suffers from a singularity in the closing flux variable, the proposed closure is regularized by a parameter β such that in the limit $\beta \to 0$, the singularity is recovered and the new closure reduces to the 5-moment closure recently put forward in [1], which reports very promising results for hypersonic shock waves. We show how the parameter β affects the Hugoniot-Locus of the equilibrium state such as to mitigate the problem of sub-shocks in the continuous shock-structure problem. Numerical examples demonstrate superior results of the new closure when compared to Grad's 5-moment closure.

References

[1] James McDonald and Manuel Torrilhon. *Affordable Robust Moment Closures for CFD Based on the Maximum-Entropy Hierarchy*. in press. 2013.