A SIMPLE WELL-BALANCED POSITIVE SOLVER FOR THE SHALLOW WATER SYSTEM

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Abstract

We introduce a numerical scheme for the shallow-water equation given by

$$\begin{cases} \partial_t h + \partial_x hu = 0 & (1) \\ \partial_t (hu) + \partial_x \left(hu^2 + \dfrac{gh^2}{2} \right) = -gh\, \partial_x b & (2) \end{cases}$$

which can be rewritten

$$\partial_t W + \partial_x F(W) = B(W) \tag{3}$$

with $W = (h,\, hu)^T$, $F(W) = \left(hu,\, hu^2 + gh^2/2\right)^T$ and $B(W) = (0,\, -gh\partial_x b)$. These equations model the evolution of the water depth and the depth-average velocity respectively h and u.

The aim of this work is to build a scheme which is positivity and entropy preserving, and well-balanced. A few schemes that endowed these properties already exist (Godunov scheme by Leroux-Chinaya-Seguin, Kinetic scheme by Perthame-Simeoni, relaxation scheme by Bouchut...) but they all are very complex to build.

This scheme is based on the solution of a simple Riemann problem with three discontinuities propagating with velocities

$$\lambda_L < 0 < \lambda_R$$

In this context, we have two intermediate states W_L^* and W_R^* defined by the integral consistency condition

$$\begin{cases} h_R\, u_R - h_L\, u_L = \lambda_R\, (h_R - h_R^*) + \lambda_L\, (h_L^* - h_L) & (4) \\ \left(h_R\, u_R^2 + \dfrac{g\, h_R^2}{2} \right) - \left(h_L\, u_L^2 + \dfrac{g\, h_L^2}{2} \right) = \lambda_R\, (h_R\, u_R - h_R^*\, u_R^*) + \lambda_L\, (h_L^*\, u_L^* - h_L\, u_L) + g\, \{h \Delta b\} & (5) \end{cases}$$

and two additional relations on the stationary wave

$$\begin{cases} h_L^*\, u_L^* = h_R^*\, u_R^* & (6) \\ h_L^* + z_L = h_R^* + z_R & (7) \end{cases}$$

to ensure the well-balanced property.

A slight modification of the scheme ensures the positivity of the water height under classical CFL condition. We are working on the proof of a fully discrete in-cell entropy inequality for the scheme.

A first test for a fluvial flow over a bump give us a first idea of the efficiency of this scheme

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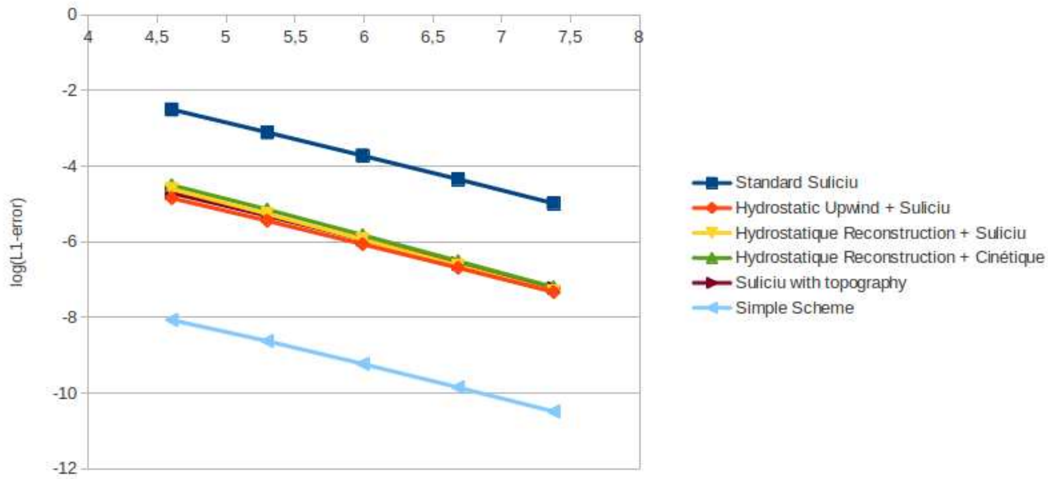

FIGURE 1 – Comparison of the L1-error for different schemes