A Multiscale Numerical Scheme for Computing Large-Scale Atmospheric Flows at High Resolution

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We present a multiscale semi-implicit numerical scheme, which is intended for the efficient computation of large-scale atmospheric flows at high resolution. Modern high-performance computing hardware is beginning to allow modelers to use grids with horizontal spacing in the range of merely a few kilometers even for planetary-scale simulations. At this resolution, small-scale balances and large-scale wave-phenomena must be accurately computed at the same time. Thus, the new scheme aims to minimize dispersion and amplitude errors in the computation of long-wave accoustic waves. While it correctly balances "slaved" dynamics of short-wave solution components induced by slow forcing, the method eliminates freely propagating compressible short-wave modes, which are under-resolved in time. This is achieved through a multilevel approach borrowing ideas from multigrid schemes for elliptic equations. The scheme is second-order accurate and admits time steps depending essentially on the flow velocity.

The basic scheme is developed in the context of the nonlinear shallow water equations, with a bottom topography varying in time. The latter simulates a source term which directly acts on the local divergence of the flow, such as latent heat release due to local condensation in atmospheric flow problems. Numerical tests verify the second-order convergence of the method and address its balancing properties for the lake at rest. Furthermore, the asymptotic multiscale regime of fast gravity waves traveling over short-range topography is considered, which is equivalent to the low Mach number regime of long-wave acoustic waves interacting with slow advection as described in Klein (*J. Comput. Phys. 121, pp. 213–237, 1995*). The numerical results of the proposed method suggest that the scheme correctly reproduces this regime, and can be therefore considered as a so-called asymptotically adaptive (or asymptotic preserving) numerical scheme.

Additionally, we show how to extend an anelastic flow solver for small-scale atmospheric problems to weakly compressible flows. This approach enables us to have a discretzation, which is asymptotically consistent in the regime of a vanishing Mach number. Preliminary results show the feasibility of the approach, and the introduction of the multiscale time-integration will enable us to simulate realistic multiscale atmospheric flow problems.