Data Assimilation for hyperbolic conservation laws: a Luenberger observer approach based on a kinetic description

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Data assimilation has become a popular strategy to refine the quality of numerical simulations of complex physical phenomena by taking into account available measurements. In particular numerous works in environmental sciences and life sciences – see [1, 2]– have used data assimilation to deal with the various sources of error restricting the performance of a numerical prediction. This is particularly the case for hyperbolic systems of conservation laws where, in the absence of dissipation, even small numerical errors are likely to propagate and expand in time.
We consider in this work the shallow water system, which is a hyperbolic and non-linear system of conservation law.
We follow the path introduced by Luenberger [3] and popularized for PDE with the *nudging* appellation [4] to define a sequential observer which converges to the actual system in one forward simulation and in which the correction term remains tractable in practice. The principle is to introduce the simplest possible correction so that the error between the actual observed trajectory and the simulated systems stabilizes rapidly to 0. However, for conservation laws, despite some recent effort [5], the observer performances remain difficult to analyze in a general context, in particular in the presence of non-linearities. In this study, we propose a new nudging strategy which, for hyperbolic conservation laws admitting a kinetic description, can be thoroughly justified even for non-linear systems.
First we explain the basic concepts of hyperbolic conservation laws and their kinetic approach. Afterwards, we expose the nudging strategy at the kinetic level on the shallow water system and emphasize the benefit of this method compared to standard nudging. We are then able to build an efficient entropy satisfying numerical assimilation scheme based on heights observations only. Eventually, we present numerical results in two dimensions.

Références

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